The Hong-Ou-Mandel experiment and Bohmian mechanics

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Here comes my discussion of the Hong-Ou-Mandel experiment. I will actually use very few equations, because a good visualization in configuration space will mostly clarify the issue.

So here is a picture to clarify my coordinate system in space and time.

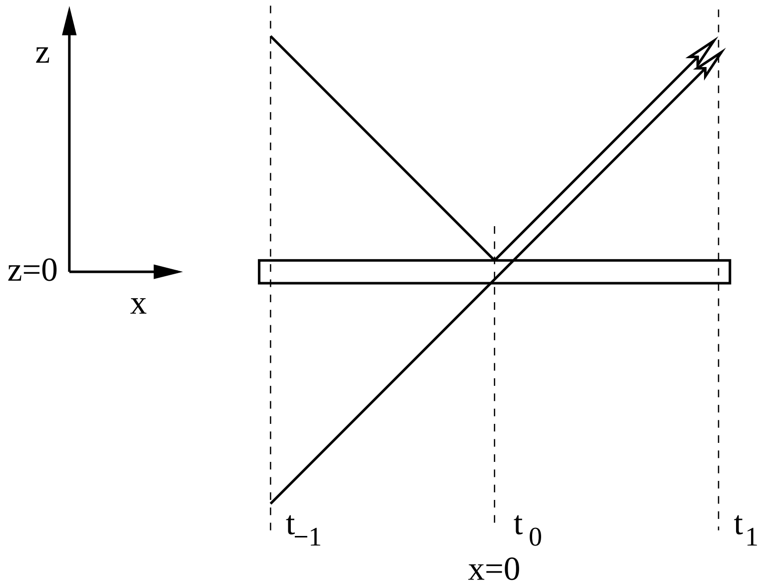

Figure 1

Now this picture is of course gleaned from a description of the Hong-Ou-Mandel experiment with *photons*. In fact, I would like to discuss the photon case, which is easy in standard quantum mechanics, but unfortunately, there are no formulas for the velocities of Bohmian particles, if the system consists of photons. Worse, I would not know how to derive them, because the relativistic Hamiltonion essentially would be a square root of a negative Laplacian, and I have no idea how to extract a divergence from that, in order to define a probability current. Using the massless limit of the Klein-Gordon equation, i.e., the wave equation instead, does not help either, because it is known that one cannot define a proper probability current for that equation.

In the atom case, the particle trajectories will be parabolic in the lab frame, not straight lines. Nevertheless, I argue that the picture can still be used. Just describe the experiment in a freely falling frame, then the atom trajectories *will* be straight lines. Moreover, the Schrödinger equation will hold in the freely falling frame to a better approximation than in the lab, because of the equivalence principle, telling us that if the frame is sufficiently small, we can forget about the influence of gravity.

In the original drawing, the rectangle depicted a beam splitter for photons, in the atom case, it should be seen as a symbolic representation of the moving optical lattice, used to make the atoms interfere. In addition, the picture suggests a symmetry that one might not suspect when viewing the experiment from the lab frame and that might be invoked *against* Bohmian mechanics. Since Bohmian mechanics will be shown to work *with* this symmetry, it will work all the better *without* it.

The interference region of the two atoms is put at the origin of the (moving) coordinate system, i.e., we measure positions relative to the interference zone.

Two identical bosonic atoms are released at some negative x value at time t_{-1}, they interact with the optical lattice near $(x, y, z) = (0, 0, 0)$ at time t_0, interfere and emerge on one side both in the same single-particle state. I have drawn them to move upward, but they could eqally well have gone downward. Their wave function will be considered at times t_{-1}, t_0, and t_1, some time after interference.

As preliminaries, let us have a look at the symmetry properties of the wave function. First, it must describe two indistinguishable bosons starting from different positions, which means it must be an entangled state that is symmetric under particle exchange:

$$\begin{aligned}\psi(x_1,y_1,z_1,x_2,y_2,z_2,t) = \frac{1}{\sqrt{2}}\Big(&\varphi_1(x_1,y_1,z_1,t)\varphi_2(x_2,y_2,z_2,t)\\ &+\varphi_2(x_1,y_1,z_1,t)\varphi_1(x_2,y_2,z_2,t)\Big)\,.\end{aligned} \tag{1}$$

The state *after* having passed the interference region could in principle be a product state, because now both bosons are in the same single-particle state, but due to the fact that the wave function must describe both possibilities of the bosons emerging at positive z values and at negative z values, it will still be a superposition of two different product states, i.e., the entanglement will continue until the photons are measured.

Another symmetry property of the wave function that could be assumed due to the symmetric setup and symmetric initial conditions, is symmetry with respect to reflection at the $z = 0$ plane.[1]

$$\psi(x_1,y_1,z_1,x_2,y_2,z_2,t) = \psi(x_1,y_1,-z_1,x_2,y_2,-z_2,t)\,. \tag{2}$$

Note that reflection changes the sign of both z arguments of the wave function. In configuration space, this symmetry corresponds to a simultaneous reflection of two coordinates, keeping the $(x_1,y_1)\times(x_2,y_2)$ "hyperplane" invariant. It is useful to immediately have a look at the consequences for the Bohmian particle velocities. We have

$$\boldsymbol{v}_k = \frac{\hbar}{m}\Im\left(\frac{\nabla_k\psi}{\psi}\right) = \frac{\hbar}{m}\Im\left(\nabla_k \ln\psi(x_1,y_1,z_1,x_2,y_2,z_2,t)\right)\,, \qquad k=1,2\,. \tag{3}$$

If ψ were a product state, the velocity of particle k would not depend on the position of the other particle, because the logarithm would be a sum of terms each depending only on the variables of a single particle, and the gradient with respect to its position variables would make the summands of all but particle k vanish. But since it is an entangled state, we do not have this simplification. The velocity of particle 1 depends on where is particle 2, and vice versa.

Let us consider what we are able to deduce about velocity components in the z direction from the symmetry (2):

$$\begin{aligned}v_z^{(1)}(x_1,y_1,0,x_2,y_2,z_2,t) &= \frac{\hbar}{m}\Im\left(\frac{\partial}{\partial z_1}\ln\psi(x_1,y_1,z_1,x_2,y_2,z_2,t)\right)\Big|_{z_1=0}\\ &= \frac{\hbar}{m}\Im\left(\frac{\partial}{\partial z_1}\ln\psi(x_1,y_1,-z_1,x_2,y_2,-z_2,t)\right)\Big|_{z_1=0}\\ &= -\frac{\hbar}{m}\Im\left(\frac{\partial}{\partial u}\ln\psi(x_1,y_1,u,x_2,y_2,-z_2,t)\right)\Big|_{u=0}\\ &= -v_z^{(1)}(x_1,y_1,0,x_2,y_2,-z_2,t)\,,\end{aligned} \tag{4}$$

from which we conclude that the z velocity component of particle 1 becomes zero on the plane $z = 0$, if particle 2 is at $z = 0$ as well, because then we have $v_z^{(1)}(\ldots,0,\ldots,0,t) = -v_z^{(1)}(\ldots,0,\ldots,0,t)$. Hence, we may infer that both particles cannot cross the $z = 0$ plane *simultaneously*. But if one of

[1] In a freely falling, i.e., local inertial frame.

them is not in the $z = 0$ plane, while the other hits that plane, the first particle will not encounter any problems passing the plane. The presence of a second particle reduces the effect of a symmetry plane from preventing *all* trajectories from passing the plane to preventing only a minority (of measure zero!) from doing so.

In the following, I would like to first discuss the behaviour of the two-particle wave function in *configuration space*. This is standard quantum mechanics and should not give rise to strong disagreement. Since I cannot easily draw a six-dimensional space, let us first get rid of uninteresting aspects. Assuming the motion to happen primarily in the xz plane, the extension of the wave function in the y directions will not display any interesting behaviour. The projection of the wave function on the y_1y_2 plane will essentially be a blob about the origin that slowly spreads. For the times considered here, it may even be possible to assume it constant. Similarly, the x_1x_2 plane will not be particularly interesting. Again just a blob, now moving at constant velocity along the bisector of the plane (and slowly getting wider). Therefore, we may focus attention on the two-dimensional subspace constituted by the z_1z_2 plane. And two dimensions are very appropriate for drawings.

Consider the picture on the right-hand side. This is a representation of the wave function at the initial time t_{-1}, a projection on the z_1z_2 plane. The green blobs are to correspond to regions containing a substantial part of the total probability of particle detection, if a position measurement were being performed. Let us say, the black boundary of the blob corresponds to a contour line of the probability density, so that in its interior, 99% of the probability are captured. Since the initial state consists of one particle at a negative z value and the other one at the opposite positive one (approximately), the blob must be roughly centered on the second bisector of the coordinate axes. Also, there must be *two* blobs, one on each side of the *first* bisector, because of the indistinguishability of the atoms, requiring symmetry of the (projected) wave function with respect to the first bisector.

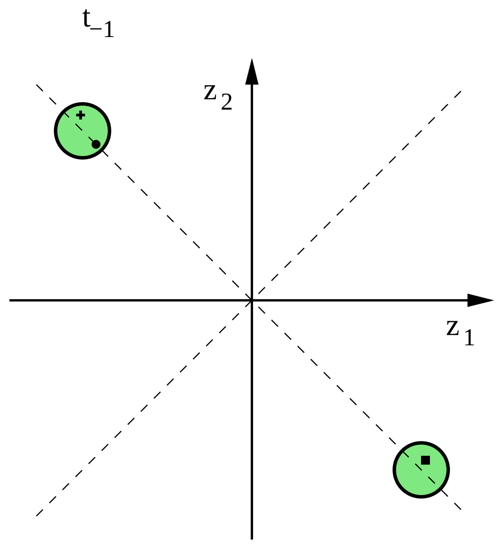

Figure 2

To begin with, we restrict ourselves to the behaviour of the wave function. How will it move? Of course, the two blobs will move towards the origin along the second bisector. The will become wider in the course of the motion which is not drawn, because the experiment can be so set up that this broadening is small.
When the wave packets reach the interference region at t_0, they essentially merge in a single blob (which will not be significantly larger than each of the original ones, it still contains 99% of the probability).
Since we know from quantum mechanics that the two particles must emerge on the same side of the $z = 0$ plane, the wave packet must continue its way along the *first* bisector. Due to the existence of both possibilities, the two atoms being able to emerge both above and below the optical grating, there must be two blobs again, see Figure 4.

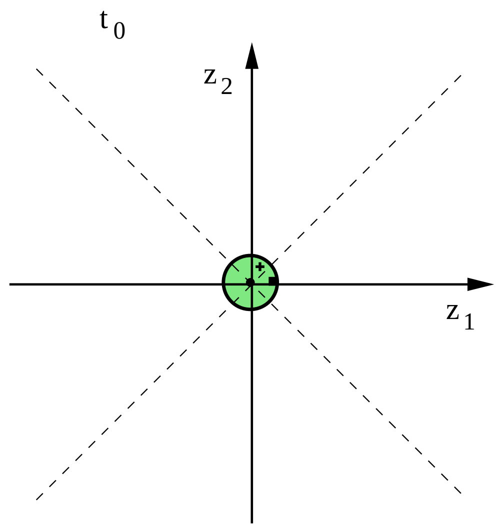

Figure 3

This image shows the situation at about t_1. Note that if the two other possibilities were quantum mechanically possible, viz. particle 1 emerging in the positive z channel and particle 2 at negative z and vice versa (particle 1 appearing at negative z values and particle 2 at positive ones), then our wave packet would have had to split into four pieces after interference, two moving out along the first bisector and two moving out along the second bisector (i.e., moving back to where the wave packets came from before interfering).

I hope so far everything is uncontroversial. This should be the qualitative behaviour of the wave function in configuration space, according to standard quantum mechanics. As we shall see, we can learn a lot from this behaviour.

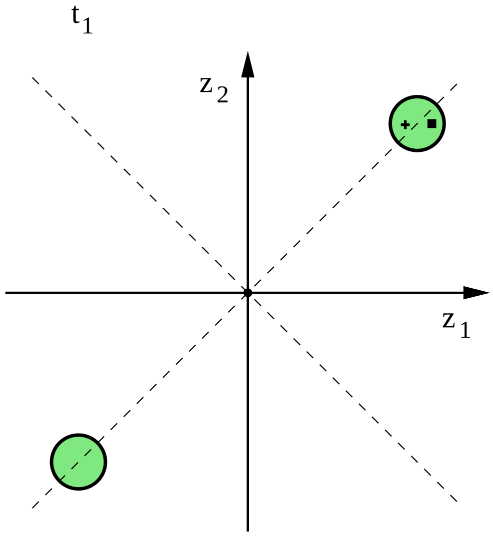

Figure 4

Now, Bohmian mechanics consists of two sections. One describes the *wave function* – by the same Schrödinger equation as standard quantum mechanics. So the *wave function* must behave precisely the same way in Bohmian mechanics as in standard non-relativistic quantum mechanics. The second section describes the dynamics of *particles*, to which we turn now.

The first thing to note is that all Bohmian particles together correspond, at a given time, to a *single point* in configuration space. The *spatial trajectories* of Bohmian particles become a *single curve* in configuration space (with an equation of motion that is even local in that space). In our case, configuration space is six-dimensional and the trajectory is a path in that six-dimensional space. At any fixed time, the projection of the six-dimensional point representing the two Bohmian particles onto the z_1z_2 plane is a single point.

The range of positions where this point can be located in our final figure is given by the two blobs – the Bohmian point must be in there with 99% probability. However, the locations where it can be according to this argument are in the first and third quadrants only, around the first bisector. This means that both Bohmian particles must be on *one* side of the $z = 0$ plane. If they were on different sides, their representative point would have to be in the second or fourth quadrant! So the wave function already tells us that the Bohmian particles must both have similar trajectories and velocities after the interference region.

To see this in more detail, let us follow some particular Bohmian particle pairs. I have drawn three of them in Figures 2 through 4. There is a cross, a square and a circle in Figure 2. The cross and the square are off the second bisector (but of course still within the wave function), the circle is precisely on it, i.e., it corresponds to a pair of Bohmian particles who have exactly opposite z positions. Moreover, I have assumed for this particular pair that both partners have the same x and y positions – otherwise there would be no guarantee that the pair would move exactly along the second bisector. Because the motion of one particle depends on the position of the other according to Eq. (3), particles with different x or y positions will not move symmetrically along the z direction, even if started from symmetric z positions. But let us assume, that for this particle pair, all the symmetry conditions are satisfied.

For the particle pair symbolized by a cross, particle 1 is at negative z in Fig. 2, particle 2 has positive z. In Fig. 3, particle 1 has crossed over to positive z. It had no problems crossing the $z = 0$ plane (i.e., the z_2 axis in the picture), because z_2 was nonzero all the time. Only simultaneous crossing is forbidden by symmetry. The two particles then both go on to the positive z side (Fig. 4).

For the particle pair symbolized by a square, particle 1 is at positve z and particle 2 at negative z. Particle 2 crosses over to positive z around time t_0 and both particles follow the wave packet that moves up and to the right along the first bisector.

Now for the particle pair depicted by a circle. Having assumed all the required symmetries, these two particles will have to follow the second bisector, meaning they will approach the origin. Since their z component of the velocity goes to zero, they will not be exactly there at t_0. They will never exactly reach the point, their trajectory has separatrix character. Hence, these two Bohmian particles will not be able to follow the wave packet up the first bisector. Therefore, I have still drawn them at the origin in Fig. 4. Is this in contradiction to standard quantum mechanics? Sure. But it does not have measurable consequences. For this to happen, two coordinate pairs of the Bohmian particles must agree and the third be opposite to each other. So the coordinates of all the Bohmian particles satisfying such a condition form a three-dimensional set. But the coordinates of the full ensemble of Bohmian particles make up a six-dimensional set. Thus, the first set has zero probability measure. A three-dimensional subset of a six-dimensional set with finite probability measure must always have zero measure, if no δ functions occur.[2] Therefore, the probability for a pair of Bohmian particles to come to rest on the $z = 0$ plane is zero. While a single counterexample may destroy a theory, this statements holds true only for counterexamples that can be made to happen with finite probability. If there are no experimentally detectable consequences, the counterexample does not work.

Note that the Bohmian particles are "metaphysically" distinguishable, because the equations tell us, deterministically, which particle has followed which trajectory. So if we knew the initial position of a particle, we could identify it forever, by following its trajectory. However, *physically* Bohmian particles are as indistinguishable as standard quantum mechanical ones. We have no way of knowing the initial position of a Bohmian particle. All the – verifiable – predictions of *probabilistic* nature are derived from the wave function in Bohmian mechanics, and the wave function has the symmetry property implying indistinguishability. On the other hand, all the *deterministic* predictions of Bohmian mechanics about particle positions, etc., are not verifiable, including distinguishability.

[2] The occurrence of δ functions would lead to illegitimate, because non-normalizable wave functions.