Superluminal communication scheme – explanation of what is wrong

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The short version is: the analysis is qualitative and thus inconclusive. A detailed quantitative analysis shows that no messages can be sent via the scheme. Nevertheless, the article is interesting, because at first sight, it is not obvious why the contraption does not work.

One point that blocked my understanding for some time is the fact that in order to compactify the notation, the full wave function was not given. Sofia presents only the spin part of the wave function of the photon and only the spatial part of the wave function of the atom, and this may detract from the actual way how nature protects herself from superluminal communication. Whenever a fundamental problem arises, it is a good idea to be as precise in notation as possible and not to omit too many details. Otherwise, the solution might be looked for in the wrong place.

For example, I believed for several days that this kind of entanglement between the *path* of an atom and the *polarization* of a photon is impossible. Even now, I am not sure that the wave function given by Sofia can be actually prepared. A slightly modified wave function would in fact allow superluminal communication, as we shall see below. However, one with the symmetries given does not.

Thus, it is important to verify that the starting assumptions are o.k.[1] In the case considered by Sofia, it appears that her premises are acceptable, after all, in spite of the *preparation device* that "magically" produces the entanglement between position and spin variables. However, it appears that the symmetry property of the spatial wave function is crucial here.[2]

The entangled wave function to be considered is

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\Big\{|\Psi_+\rangle\,|H\rangle + |\Psi_-\rangle\,|V\rangle\Big\}\,. \tag{1}$$

Herein, $|\Psi_+\rangle$ and $|\Psi_-\rangle$ describe spatial parts of the wave function of the atom that is sent through an interferometer on Alice's side (and the spin part of its wave function has been suppressed), whereas the states $|H\rangle$ and $|V\rangle$ refer to horizontally and vertically polarized states of the photon (the position part of whose wave function has been suppressed), later to be caught by Bob. We take Sofia's word that the experimentalists are able to produce such a state.

The symmetry of the apparatus then imposes[3]

$$\langle \boldsymbol{r}|\Psi_+(t)\rangle = \Psi_+(x,y,z,t) = \Psi_-(x,y,-z,t) = \langle \boldsymbol{r}_\perp|\Psi_-(t)\rangle\ . \tag{2}$$

All states are normalized: $\langle\Psi_+|\Psi_+\rangle = \langle\Psi_-|\Psi_-\rangle = \langle H|H\rangle = \langle V|V\rangle = 1$. ($\langle H|V\rangle = 0$.)

Moreover, we will need the diagonally and antidiagonally polarized states of the photon

$$|D\rangle = \frac{1}{\sqrt{2}}\Big(|H\rangle + |V\rangle\Big)\,, \qquad |A\rangle = \frac{1}{\sqrt{2}}\Big(|H\rangle - |V\rangle\Big)\,,$$

$$\Rightarrow\quad |H\rangle = \frac{1}{\sqrt{2}}\Big(|D\rangle + |A\rangle\Big)\,, \qquad |V\rangle = \frac{1}{\sqrt{2}}\Big(|D\rangle - |A\rangle\Big)\,. \tag{3}$$

[1] This serves mostly to point out that by starting from impossibilities, we may seem to be able to achieve other impossibilities and will not find a mistake in the *chain of arguments*.

[2] In an earlier version, Sofia used a polarizing beam splitter, a device that everybody with a bit of optics education can understand. To obtain the effect, a $\lambda/2$-platelet was used to rotate the polarization by 90 degrees in one arm of the interferometer. This was also good, because it appeared that it is this piece of equipment that produces the interesting behavior. How it works, is again easy to unterstand, so there is no magic involved in preparing the wave function. Analysis could focus on a well-accepted type of wave function and with this setup I might have found the solution a few days earlier, not wasting my time in trying to show that with the current setup additional impossibilities are involved. Since I believe the two-photon version was not posted here, I will give the solution for the situation actually discussed.

[3] $\boldsymbol{r}_\perp$ is the mirror image of $\boldsymbol{r}$ with respect to the $z = 0$ plane.

The experiment consists in sending the atom through the interferometer, with the wave packets on Alice's side first separating and then approaching again to overlap in some interference region. Since the amplitudes Ψ_+ and Ψ_- are identical along $z = 0$, Alice will, if she measures the atom in the $z = 0$ plane, collapse Bob's photon wave function on $|D\rangle$. This would seem, if Bob is in space-like separation from Alice, to imply that she could send him a faster-than-light signal by either measuring or not measuring the atom, as long as her measurement is made sufficiently close to the $z = 0$ plane. Bob would measure the polarization of his photon in the A/D basis and conclude from getting $|D\rangle$ more often than $|A\rangle$ that in these cases Alice has done the measurement, whereas if he does not make such an observation, Alice has not done a measurement. This would allow to send bits 1 and 0 probabilistically. Each bit would require a sufficient number of photons measured to be able to say it was a 1 or a 0 within a certain confidence interval.

Let us see whether this qualitative argument is borne out by calculation. It will be useful to rewrite the wave function (1) in terms of the A/D basis:

$$\begin{aligned} |\Psi(t)\rangle &= \frac{1}{\sqrt{2}}\Big\{|\Psi_+\rangle \frac{1}{\sqrt{2}}\left(|D\rangle + |A\rangle\right) + |\Psi_-\rangle \frac{1}{\sqrt{2}}\left(|D\rangle - |A\rangle\right)\Big\} \\ &= \frac{1}{\sqrt{2}}\Big\{\frac{1}{\sqrt{2}}\left(|\Psi_+\rangle + |\Psi_-\rangle\right)|D\rangle + \frac{1}{\sqrt{2}}\left(|\Psi_+\rangle - |\Psi_-\rangle\right)|A\rangle\Big\} \\ &= \frac{1}{\sqrt{2}}\Big\{|\Psi_e\rangle\,|D\rangle + |\Psi_o\rangle\,|A\rangle\Big\}\,, \end{aligned} \tag{4}$$

where the position representations of $|\Psi_e\rangle$ and $|\Psi_o\rangle$ are given by

$$\Psi_e(\boldsymbol{r},t) = \frac{1}{\sqrt{2}}\left(\Psi_+(\boldsymbol{r},t) + \Psi_-(\boldsymbol{r},t)\right)\,, \qquad \Psi_o(\boldsymbol{r},t) = \frac{1}{\sqrt{2}}\left(\Psi_+(\boldsymbol{r},t) - \Psi_-(\boldsymbol{r},t)\right)\,, \tag{5}$$

and we obviously have, because of the symmetry property (2):

$$\begin{aligned} \Psi_e(x,y,z,t) &= \frac{1}{\sqrt{2}}\left(\Psi_+(x,y,z,t) + \Psi_+(x,y,-z,t)\right) = \Psi_e(x,y,-z,t)\,, \\ \Psi_o(x,y,z,t) &= \frac{1}{\sqrt{2}}\left(\Psi_+(x,y,z,t) - \Psi_+(x,y,-z,t)\right) = -\Psi_o(x,y,-z,t)\,, \end{aligned} \tag{6}$$

that is, $\Psi_e(x,y,z,t)$ is an *even* function of z and $\Psi_o(x,y,z,t)$ and *odd* one.

Let us first find out what kind of statistics Bob will measure, if Alice does *not* make any measurements. This can be determined by taking the density operator that describes the two-particle system and tracing out the states of Alice's particle to get Bob's single-particle density operator. Of course, the density operator of the *two-particle* system describes a *pure* state

$$\rho = |\Psi(t)\rangle\,\langle\Psi(t)| = \frac{1}{2}\Big\{|\Psi_e\rangle\,|D\rangle + |\Psi_o\rangle\,|A\rangle\Big\}\Big\{\langle\Psi_e|\,\langle D| + \langle\Psi_o|\,\langle A|\Big\}\,, \tag{7}$$

from which we get the density operator of Bob's photon using position eigenstates to do the trace

$$\begin{aligned} \rho_B = tr_A(\rho) &= \frac{1}{2}\int \mathrm{d}^3 r\Big\{\langle\boldsymbol{r}|\Psi_e\rangle\,|D\rangle + \langle\boldsymbol{r}|\Psi_o\rangle\,|A\rangle\Big\}\Big\{\langle\Psi_e|\boldsymbol{r}\rangle\,\langle D| + \langle\Psi_o|\boldsymbol{r}\rangle\,\langle A|\Big\} \\ &= \frac{1}{2}\int \mathrm{d}^3 r\Big\{\Psi_e(\boldsymbol{r})\,|D\rangle + \Psi_o(\boldsymbol{r})\,|A\rangle\Big\}\Big\{\Psi_e^*(\boldsymbol{r})\,\langle D| + \Psi_o^*(\boldsymbol{r})\,\langle A|\Big\} \\ &= \frac{1}{2}\bigg(|D\rangle\,\langle D|\int \mathrm{d}^3 r\,|\Psi_e(\boldsymbol{r})|^2 + |A\rangle\,\langle A|\int \mathrm{d}^3 r\,|\Psi_o(\boldsymbol{r})|^2 \\ &\qquad + |A\rangle\,\langle D|\int \mathrm{d}^3 r\,\Psi_o(\boldsymbol{r})\Psi_e^*(\boldsymbol{r}) + |D\rangle\,\langle A|\int \mathrm{d}^3 r\,\Psi_e(\boldsymbol{r})\Psi_o^*(\boldsymbol{r})\bigg)\,. \end{aligned} \tag{8}$$

Since $|\Psi_e\rangle$ and $|\Psi_o\rangle$ are normalized, too, the first two integrals of the last equality must both be equal to one. Let us denote

$$2m \equiv \int \mathrm{d}^3 r\, \Psi_o(\boldsymbol{r})\Psi_e^*(\boldsymbol{r})\,, \tag{9}$$

then Bob's density operator reads

$$\begin{aligned}\rho_B &= \frac{1}{2}\left(|D\rangle\langle D| + |A\rangle\langle A|\right) + m\,|A\rangle\langle D| + m^*\,|D\rangle\langle A| \\ &= \frac{1}{2} + m\,|A\rangle\langle D| + m^*\,|D\rangle\langle A|\ .\end{aligned} \tag{10}$$

Assume for the time being that the overlap integral m is nonzero and, as is to be expected for arbitrary spatial wave functions, a function of time. Then there is a simple scheme for superluminal communication: Any measurement of the atom position on Alice's side effectively fixes the wave function of Bob's photon until he makes a measurement, because any superposition of $|D\rangle$ and $|A\rangle$ is an eigenstate to the Hamiltonian.[4] So all Alice has to do is to make a position measurement at time Δt_1 [after preparation of the state (1)], if she wants to send a bit 1 and at time Δt_2 [with $m(\Delta t_2) \neq m(\Delta t_1)$], if she wants to send a bit 0. Bob measures his photon at some time $> \max(\Delta t_1, \Delta t_2)$, and since Alice's measurement effectively freezes his density operator at either of the m values, he can distinguish these two cases by making a sufficient number of measurements, which may be realized by redundancy, i.e., by Alice repeating each experiment with measurement at Δt_1 or Δt_2 often enough to obtain a desired conficence interval.[5] Using error-correcting codes, the success probability for transmission of the message may be further increased. Moreover, Bob is able to find out whether Alice has finished sending, if $m(\infty) \neq m(\Delta t_{1/2})$.

Alas, this scheme does not work under the conditions introduced, because m *turns out to be zero*. Since Ψ_o is an odd function of z and Ψ_e an even one, the product $\Psi_o(\boldsymbol{r})\Psi_e^*(\boldsymbol{r})$ is odd.[6] Hence the integration over z makes the integral (9) vanish. We then end up with

$$\rho_B = \frac{1}{2}\,, \tag{11}$$

i.e., ρ_B is proportional to the identity operator. No matter in what basis Bob measures polarizations, he will always measure half of the photons in one basis state and the other half in the state orthogonal to it. In particular, this is true for the A/D basis:

$$p(|D\rangle) = \frac{1}{2}\,. \tag{12}$$

Let us now check, whether a measurement on Alice's side slightly before Bob does his measurement,[7] will change the statistics of his measurement results.

Suppose Alice performs a position measurement on state $|\Psi(t)\rangle$. This will project the state onto an eigenstate of the position operator,[8] labeled by the measured position $\boldsymbol{r}_m$:

$$|\varphi_A\rangle = \frac{1}{\sqrt{2}}\,|\boldsymbol{r}_m\rangle\langle\boldsymbol{r}_m|\Big\{|\Psi_e\rangle\,|D\rangle + |\Psi_o\rangle\,|A\rangle\Big\} = \frac{1}{\sqrt{2}}\,|\boldsymbol{r}_m\rangle\Big\{\Psi_e(\boldsymbol{r}_m)\,|D\rangle + \Psi_o(\boldsymbol{r}_m)\,|A\rangle\Big\}\,. \tag{13}$$

[4] The photon travels, before being measured, through free space without any polarization changing equipment.

[5] This is why I believe that arbitrary spatial wave functions cannot be entangled with spins or polarizations. In the general case, m will be time dependent.

[6] Complex conjugation does not affect the propery of evenness or oddness.

[7] I will not go into the ramifications of a relativistic description and simultaneity issues. We are doing non-relativistic Schrödinger theory here and consider only a single synchrony. All that is used from relativity is the causality structure, saying that space-like events cannot act causally onto each other. This implies the assumption that local operators acting on space-like states commute.

[8] This is an improper Hilbert vector, "normalized via a δ function", i.e., we are working in a rigged Hilbert space. The discussion could be made mathematically cleaner by introducing eigen differentials normalized to one, but we will use the formally simpler approach employing Dirac vectors.

Since this is a product state, we can restrict ourselves to the factor from the photon Hilbert space in calculating Bob's probability density to measure $|D\rangle$ immediately after:[9]

$$w(|D\rangle)_{|\varphi_A\rangle} = \left| \langle D| \frac{1}{\sqrt{2}} \Big\{ \Psi_e(\boldsymbol{r}_m) \,|D\rangle + \Psi_o(\boldsymbol{r}_m) \,|A\rangle \Big\} \right|^2 = \frac{1}{2} \left|\Psi_e(\boldsymbol{r}_m)\right|^2 . \tag{14}$$

This is the joint probability for Alice measuring $|\boldsymbol{r}_m\rangle$ and Bob measuring $|D\rangle$ afterwards.[10] Actually, it is a probability density, because $\boldsymbol{r}_m$ is continuous and $|\Psi_e(\boldsymbol{r}_m)|^2$ has the dimension of an inverse volume.

However, Alice has no control over what position she will measure; probabilities for different positions will be determined by the wave function. Neither will Bob know what position Alice has measured. In many repetitions of the experiment with both of them doing their measurements at the same time offset relative to the initial preparation of the state (1), Bob will collect statistics over all measurements of Alice. Since different positions are independent, probabilities simply add up, i.e., the probability density integrates up:

$$p(|D\rangle) = \int w(|D\rangle)_{|\varphi_A\rangle} \mathrm{d}^3 r_m = \frac{1}{2} \int \left|\Psi_e(\boldsymbol{r}_m)\right|^2 \mathrm{d}^3 r_m = \frac{1}{2} . \tag{15}$$

This agrees with Eq. (12), meaning that Bob obtains the same statistics for measurements of state $|D\rangle$, whether or not Alice performs position measurements on her atoms before his photon measurements.

However, we can integrate over all of space only, if Alice measures the position of the atom in all experiments.[11] Let us see now what happens, if Alice restricts her position measurements to a strip $I = [-z_0, z_0]$ about the $z = 0$ plane. She can do that easily, by keeping her particle detector in the interval I, thus missing all atoms that are not in the interval at the time of measurement.[12] What changes is that we must not integrate over all of space now but have to restrict integration in the z direction:

$$p(|D\rangle \wedge I) = \frac{1}{2} \int_{-\infty}^{\infty} \mathrm{d}x_m \int_{-\infty}^{\infty} \mathrm{d}y_m \int_{-z_0}^{z_0} \mathrm{d}z_m \left|\Psi_e(\boldsymbol{r}_m)\right|^2 < \frac{1}{2} . \tag{16}$$

So the joint probability for Alice measuring the atom in I and Bob measuring polarization D is smaller than $1/2$. However, Bob cannot notice this, because now he will also measure some photons for which Alice has missed the atom on the other side.

What is the probability for these to be found at polarization D? Alice still does her measurement attempt before Bob. If she misses the atom, she has nevertheless done a (very rough) position measurement[13] having determined the atom to be in the complement $C(I) = (-\infty, -z_0) \cup (z_0, \infty)$

[9] Otherwise we might consider, instead of projections on single position eigenstates, projectors on finite position intervals such as $\int_{|\boldsymbol{r}-\boldsymbol{r}_m|<\Delta r} \mathrm{d}^3 r \,|\boldsymbol{r}\rangle \langle \boldsymbol{r}|$.

[10] If we had wanted the conditional probability instead, we would have had to normalize Bob's factor of state (13) first. However, conditional probabilities referring to different conditions cannot be added, so we go for joint probabilities instead.

[11] Moreover assuming somewhat unrealistically that all position measurements are successful.

[12] She will in fact miss more than those, if her detector is not capable of catching the atom no matter what are its x and y positions. For the purpose of the argument let us forget about this complication. Her detector could be an extended slab of absorbing material of thickness $2z_0$, covering all of the area in which the atom could possibly appear at detection time. The absorption probability should be one and Alice should be able to move the slab into and out of detection position at will in each run of the experiment.

[13] Supposing her detector has 100% efficiency.

of the considered slab about the $z=0$ plane. This measurement must collapse the wave function, too, by projecting the state (1) via

$$\int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}y \left(\int_{-\infty}^{-z_0} + \int_{z_0}^{\infty}\right) \mathrm{d}z \, |\boldsymbol{r}\rangle \langle \boldsymbol{r}| = 1 - \int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}y \int_{-z_0}^{z_0} \mathrm{d}z \, |\boldsymbol{r}\rangle \langle \boldsymbol{r}| \ , \tag{17}$$

whence

$$\begin{aligned} |\varphi_{\neg A}\rangle &= \left(1 - \int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}y \int_{-z_0}^{z_0} \mathrm{d}z \, |\boldsymbol{r}\rangle \langle \boldsymbol{r}|\right) \frac{1}{\sqrt{2}} \Big\{ |\Psi_e\rangle \, |D\rangle + |\Psi_o\rangle \, |A\rangle \Big\} \ , \\ &= \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}y \left(\int_{-\infty}^{\infty} - \int_{-z_0}^{z_0}\right) \mathrm{d}z \Big\{ \Psi_e(\boldsymbol{r}) \, |D\rangle + \Psi_o(\boldsymbol{r}) \, |A\rangle \Big\} \, |\boldsymbol{r}\rangle \end{aligned} \tag{18}$$

and Bob's probability of measuring polarization D for his photon becomes:

$$\begin{aligned} p(|D\rangle \wedge |\phi_{\neg A}\rangle &\equiv p(|D\rangle \wedge C(I)) \\ &= \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} \mathrm{d}x' \int_{-\infty}^{\infty} \mathrm{d}y' \left(\int_{-\infty}^{\infty} - \int_{-z_0}^{z_0}\right) \mathrm{d}z' \, \langle \boldsymbol{r}' | \, \Psi_e^*(\boldsymbol{r}') \\ &\quad \times \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}y \left(\int_{-\infty}^{\infty} - \int_{-z_0}^{z_0}\right) \mathrm{d}z \, \Psi_e(\boldsymbol{r}) \, |\boldsymbol{r}\rangle \\ &= \frac{1}{2} \int \mathrm{d}x' \int \mathrm{d}x \int \mathrm{d}y' \int \mathrm{d}y \left(\int_{-\infty}^{\infty} - \int_{-z_0}^{z_0}\right) \mathrm{d}z' \\ &\quad \left(\int_{-\infty}^{\infty} - \int_{-z_0}^{z_0}\right) \mathrm{d}z \, \delta(\boldsymbol{r}' - \boldsymbol{r}) \Psi_e^*(\boldsymbol{r}') \Psi_e(\boldsymbol{r}) \\ &= \frac{1}{2} \int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}y \left(\int_{-\infty}^{\infty} - \int_{-z_0}^{z_0}\right) \mathrm{d}z \ |\Psi_e(\boldsymbol{r})|^2 \end{aligned} \tag{19}$$

Renaming the integration variables in (16) by dropping the subscript m (signifying that $\boldsymbol{r}_m$ is a measured value), we find

$$\begin{aligned} p(|D\rangle) &= p(|D\rangle \wedge I) + p(|D\rangle \wedge C(I)) \\ &= \frac{1}{2} \int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}y \int_{-z_0}^{z_0} \mathrm{d}z \, |\Psi_e(x,y,z)|^2 \\ &\quad + \frac{1}{2} \int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}y \left(\int_{-\infty}^{\infty} - \int_{-z_0}^{z_0}\right) \mathrm{d}z \, |\Psi_e(x,y,z)|^2 \\ &= \frac{1}{2} \int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}y \int_{-\infty}^{\infty} \mathrm{d}z \, |\Psi_e(x,y,z)|^2 = \frac{1}{2} \int |\Psi_e(\boldsymbol{r})|^2 \, \mathrm{d}^3 r = \frac{1}{2} \, . \end{aligned} \tag{20}$$

Once again, Bob cannot make out from the statistics of his observations what Alice has done. He finds the state $|D\rangle$ with probability $1/2$ and the state $|A\rangle$ with the same probability, assuming he catches all the photons. If he misses some photons, then under a fair sampling assumption,[14] he will still detect both polarization directions with the same probability, although this probability is now smaller than $1/2$, depending on the probability of failure of detection.

Finally, let me consider how the results from Sofia's paper can be reconciled with this. I use a general spatial wave function with only the symmetry properties imposed, whereas Sofia gives an explicit (approximate) form of the wave function in terms of Gaussian wave packets. One approximation used is that the packet moves without dispersion, i.e., it does not spread. This should not be critical

[14] Meaning that his failure to detect a photon is independent of its polarization.

for sufficiently short time intervals. Sofia then calculates, in her words "the ratio of the probabilities for obtaining the two polarizations of the photon" (on Bob's side):

$$\frac{\mathsf{Prob}(z,t,A)}{\mathsf{Prob}(z,t,D)} = \frac{\sinh^2\left[zu_+(t)/(2\sigma_z)^2\right] + \sin^2\left(p_{0,z}/\hbar\right)}{\sinh^2\left[zu_+(t)/(2\sigma_z)^2\right] + \cos^2\left(p_{0,z}/\hbar\right)}\,, \tag{21}$$

where $u_+(t)$ is the position of the center of wave packet $\Psi_+(\boldsymbol{r},t)$ and $p_{0,z}$ is the z component of the momentum associated with the plane wave prefactor of the Gaussian. While this is a ratio of probability densities rather than probabilities, the formula may be used to visualize what is happening. The ratio is smaller than one as long as the absolute value of the sine is smaller than that of the cosine, so if we want it to be smaller than one in the whole strip, in which Alice is to do her measurements, that strip should not be wider than $2z_0 = \pi\hbar/(2p_{0,z})$, as Sofia remarks herself. This is a quarter of the de Broglie wavelength associated with the motion along the z direction.[15] Therefore, the width of a strip, for which Bob unambiguously obtains a higher probability of polarization D than of polarization A is normally pretty small, small enough that the atom even can tunnel through it with nonnegligible probability. Hence, it is clear that if the strip is made that small, Alice will not detect atoms with probability close to one.

The strip can be made wider, if the angle at which atoms arrive is made smaller, increasing the trace wavelength. This means that $p_{0,z}$ decreases, which increases the uncertainty in the atom's position in the z direction, reducing Alice's detection probability in a countereffect to a possible increase due to the larger strip width. Photons measured by Bob with non-detected atoms on Alice's side will compensate the overshoot in probability of D over A.

If, in order to increase Alice's detection chances, the strip is made wider *without* changing the propagation directions of the wave packets, the ratio between the two probability densities will approach one more closely (because the hyperbolic sine term increases), so that fewer missed atoms are needed to compensate any disequilibrium.

So having only the result (21), the situation is by no means clear, because it estimates only part of the effects on Bob's detection probability (not taking into account Alice's failures to measure an atom) and it is not difficult to give arguments towards a reduction of any deviation from equal probabilities for both polarizations.

Hence, while (21) may suggest a surprising possibility of superluminal communication by the method considered, the result is not conclusive.

This is rectified by the present calculations for all situations in which Alice's and Bob's measurements are instantaneous (and at fixed times). For that case I have shown, that Bob's overall chance of measuring polarization D is exactly equal to that of measuring A, so that no scheme for superluminal communication can be construed this way. What has not been looked at in detail is the possibility that Alice's measurement will take some time (depending on the size of z_0) and that it is not performed at the same time (offset) in all runs. It is more difficult to discuss these situations – in the first, either the collapse of the wave function must become a prolonged process somehow or the interaction before the final collapse must be modeled. The spatial wave function will change while its center approaches $z = 0$, rendering detailed calculations difficult. I doubt, however, that accurate calculations will reveal any chances for the scheme to work.

[15] The de Broglie wavelength of the atom is somewhat shorter, because it is defined as the distance of wave maxima along a direction that is perpendicular to the surfaces of constant phase. $2\pi\hbar/p_{0,z}$ is a trace wavelength, measured at an angle to these surfaces, i.e., parallel to the z axis.