John S. Bell's concept of local causality

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John Stewart Bell's famous theorem is widely regarded as one of the most important developments in the foundations of physics. Yet even as we approach the 50th anniversary of Bell's discovery, its meaning and implications remain controversial. Many workers assert that Bell's theorem refutes the possibility suggested by Einstein, Podolsky, and Rosen (EPR) of supplementing ordinary quantum theory with "hidden" variables that might restore determinism and/or some notion of an observer-independent reality. But Bell himself interpreted the theorem very differently—as establishing an "essential conflict" between the well-tested empirical predictions of quantum theory and relativistic local causality. Our goal is to make Bell's own views more widely known and to explain Bell's little-known formulation of the concept of relativistic local causality on which his theorem rests. We also show precisely how Bell's formulation of local causality can be used to derive an empirically testable Bell-type inequality and to recapitulate the EPR argument.
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I. INTRODUCTION

In its most general sense, "local causality" is the idea that physical influences propagate continuously through space—that what Einstein famously called "spooky actions at a distance" are impossible.[1] In addition to originating this catchy phrase, Einstein was chiefly responsible for the relativistic sense of local causality, according to which causal influences should not only propagate continuously (never hopping across a gap in which no trace is left) but also do so always at the speed of light or slower. The elaboration and formulation of this idea will be our central concern.

The pre-relativistic "no action at a distance" sense of local causality has played an important role in the construction and assessment of theories throughout the history of physics.[2,3] For example, some important objections to Newton's theory of gravitation centered on the theory's alleged positing of non-local action at a distance. Newton's own view seems to have been that although his theory claimed (for example) that the Sun exerted causal influences on the distant planets, this influence was consistent with local causality, which he strongly endorsed:

> "It is inconceivable that inanimate brute matter should, without the mediation of something else which is not material, operate upon and affect other matter without mutual contact… That gravity should be innate, inherent, and essential to matter, so that one body may act upon another at a distance through a vacuum, without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity that I believe no man who has in philosophical matters a competent faculty of thinking can ever fall into it."[4]

Newton's idea was evidently that his gravitational theory didn't provide a complete description of the underlying (and presumably local) mechanism "by… which [massive bodies'] action and force may be conveyed from one to another."[5]

Such debates had a philosophical character because at the time nothing was unambiguously excluded by the requirement of locality. Any apparent action at a distance in a theory could be rendered compatible with local causality by following Newton and by denying that the theory in question provided a complete description of the relevant phenomena.

This changed in 1905 with Einstein's discovery of special relativity, which for the first time identified a class of causal influences—those that propagate faster than light—as inconsistent with local causality. As Einstein explained,

> "The success of the Faraday-Maxwell interpretation of electromagnetic action at a distance resulted in physicists becoming convinced that there are no such things as instantaneous action at a distance (not involving an intermediary medium) of the type of Newton's law of gravitation. According to the theory of relativity, action at a distance with the velocity of light always takes the place of instantaneous action at a distance or of action at a distance with an infinite velocity of transmission. This is connected with the fact that the velocity c plays a fundamental role in the theory."[6]

The speed of light c plays a fundamental role regarding causality because of the relativity of simultaneity. For two events A and B with space-like separation (that is, such that a signal connecting A and B would have to propagate faster than c), the time ordering is ambiguous: different inertial observers will disagree about whether A precedes B in time or vice versa. According to special relativity, there is no objective fact about which event occurs first, and hence no possibility of a causal relation between them, because the relation between a cause and its effect is necessarily time-asymmetric. As Bell explained, "To avoid causal chains going backward in time in some frames of reference, we require them to go slower than light in any frame of reference."[7,8]

After the advent of special relativity, the relativistic sense of local causality was soon used to critique other developing theories, much as the pre-relativistic concept had been used against Newton's theory. Indeed, it was Einstein himself—in both the Einstein, Podolsky, and Rosen (EPR) paper[9] and several related but less widely known arguments[10]—who first pointed out that Copenhagen quantum theory violated special relativity's locality constraint. According to Einstein, that theory's account of measurement combined with Bohr's completeness doctrine committed the theory to the sort of non-local causation which was, according to Einstein, prohibited by special relativity. Einstein thus rejected Bohr's completeness doctrine and supported something like what is now (unfortunately[11]) called the local "hidden variables" program.

Note the parallel to Newtonian gravity, with the non-locality in a candidate theory being rendered as either real or merely apparent, depending on whether or not we interpret the theory as providing a complete description of the physical processes in question. Einstein's assessment of Copenhagen quantum theory with regard to local causality thus parallels Newton's analysis of his own theory of gravitation: the theory, if regarded as complete, violates locality, and hence upholding locality requires denying completeness.

This brings us to the main subject of the paper: the work of J. S. Bell. Bell accepted Einstein's proof of the non-locality of Copenhagen quantum theory. In particular, Bell accepted as valid "the EPR argument *from locality to* deterministic hidden variables."[13] This argument involves a pair of specially prepared particles that are allowed to separate to remote locations. An observation of some property of one particle permits the observer to learn something about a corresponding property of the distant particle. According to the Copenhagen view, the distant particle fails to possess a definite value for the property in question prior to the observation; it is precisely the observation of the nearby particle which—in apparent violation of local causality—triggers the crystallization of this newly real property for the distant particle.

In Bell's recapitulation of the argument, for EPR this

> "showed that [Bohr, Heisenberg, and Jordan] had been hasty in dismissing the reality of the microscopic world. In particular, Jordan had been wrong in supposing that nothing was real or fixed in that world before observation. For after observing only one particle the result of subsequently observing the other (possibly at a very remote place) is immediately predictable. Could it be that the first observation somehow fixes what was unfixed, or makes real what was unreal, not only for the near particle but also for the remote one? For EPR that would be an unthinkable 'spooky action at a distance.' To avoid such action at a distance [one has] to attribute, to the space-time regions in question, *real* properties in advance of observation, correlated properties, which *predetermine* the outcomes of these particular observations. Because these real properties, fixed in advance of observation, are not contained in [the] quantum formalism, that formalism for EPR is *incomplete*. It may be correct, as far as it goes, but the usual quantum formalism cannot be the whole story."[13]

Bell thus agreed with Einstein that the local hidden variables program constituted the only hope for a locally causal re-formulation of quantum theory. Bell's historic contribution was a theorem establishing that no such local hidden variable theory—and hence no local theory of any kind—could generate the correct empirical predictions for a certain class of experiments.[14] According to Bell, we must therefore accept the real existence of faster-than-light causation and hence an apparent conflict with the requirements of special relativity: "For me then this is the real problem with quantum theory: the apparently essential conflict between any sharp formulation and fundamental relativity. That is to say, we have an apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory…."[15]

Bell even suggested, in response to his theorem and relevant experiments,[16,17] the rejection of "fundamental relativity" and the return to a Lorentzian view in which there is a dynamically privileged, though probably empirically undetectable, reference frame: "It may well be that a relativistic version of [quantum] theory, while Lorentz invariant and local at the observational level, may be necessarily non-local and with a preferred frame (or aether) at the fundamental level."[18] And elsewhere he remarked:

> "… I would say that the cheapest resolution is something like going back to relativity as it was before Einstein, when people like Lorentz and Poincaré thought that there was an aether—a preferred frame of reference—but that our measuring instruments were distorted by motion in such a way that we could not detect motion through the aether. Now, in that way you can imagine that there is a preferred frame of reference, and in this preferred frame of reference things do go faster than light…. Behind the apparent Lorentz invariance of the phenomena, there is a deeper level which is not Lorentz invariant… [This] pre-Einstein position of Lorentz and Poincaré, Larmor and Fitzgerald, was perfectly coherent, and is not inconsistent with relativity theory. The idea that there is an aether, and these Fitzgerald contractions and Larmor dilations occur, and that as a result the instruments do not detect motion through the aether–that is a perfectly coherent point of view."[19,20]

Our intention is not to argue for this radical view, but to explain Bell's rationale for contemplating it. This rationale involves a complex chain of reasoning involving at least these four steps: (1) arguing that special relativity prohibits causal influences between space-like separated events, (2) constructing a precise formulation of this prohibition, that is, of relativistic local causality, (3) deriving of an empirically testable inequality from this formulation of local causality, and (4) establishing that the inequality is inconsistent with empirical data.

There is an extensive literature in which each of these steps is subjected to a critical analysis. The time-asymmetric character of causal relations, which was used in the argument for (1) that we have sketched, has, for example, been challenged by Price[21] and (in a very different way, based on earlier work by Bell[22]) Tumulka.[23] And there remain loopholes in the experiments which test Bell's inequality, such that one might conceivably doubt claim (4).[24] But for the most part, physicists do not seriously question (1) and regard (4) as having been established with reasonable conclusiveness. The controversies about the meaning and implications of Bell's theorem have thus centered on (2) and (3).

But what is said about (3)—the question of whether and how a Bell-type inequality is entailed by local causality—depends on whether and how (2) has been addressed. And sadly, Bell's own views on (2) have been almost invisible in the literature. (Ref. 24, for example, does not acknowledge Bell's formulation of local causality, and instead proposes an alternative formulation very different from Bell's.) It is thus not surprising that many have summarized the implications of Bell's theorem in ways very different from Bell's own. Usually, it is claimed that Bell's inequality follows not from local causality alone, but from the conjunction of local causality with some additional assumption such as "realism" or "determinism." One or more of these assumptions, rather than relativistic local causality, is then typically blamed for the inconsistency with experiment.[25–32]

The bulk of our discussion will focus on Bell's formulation of local causality, that is, his views on (2). This discussion will be based primarily on Ref. 7, published in 1990, the same year as his untimely death. Explaining Bell's formulation of locality will require also sketching Bell's interesting and refreshingly unorthodox views on several related issues in the foundations of quantum theory. The discussion will be elaborated and supported with excerpts from Bell's many other papers.

The main audience for the paper is readers with little or no prior knowledge of Bell's theorem beyond what they have read in textbooks. Almost all of the issues raised are discussed because some kind of misunderstanding or ignorance of them is present in the literature. We will provide occasional citations to works that exemplify the various important misunderstandings. But length considerations and the desire to keep the paper self-contained do not allow any extensive polemical discussions.

The paper is organized as follows. In Sec. II, we jump quickly from some of Bell's preliminary, qualitative statements to his quantitative formulation of relativistic local causality. In Secs. III–V, we clarify some controversial or unfamiliar terms that appear in Bell's formulation and contrast them to other ideas with which they have sometimes been confused. Section VI shows how local causality as formulated by Bell can be used to derive an empirically testable Bell-type inequality, and how it can be used to recapitulate the EPR argument in a rigorous way. In Sec. VII, we will summarize the arguments presented and acknowledge some of the limitations and open questions regarding Bell's formulation.

II. LOCAL CAUSALITY: OVERVIEW

We begin with a qualitative formulation of Bell's concept of local causality. In answer to an interview question about the meaning of locality, Bell responded:[33]

> "It's the idea that what you do has consequences only nearby, and that any consequences at a distant place will be weaker and will arrive there only after the time permitted by the velocity of light. Locality is the idea that consequences propagate continuously, that they don't leap over distances."[34]

Bell gave a more careful but still qualitative formulation of what he called the "principle of local causality" in 1990: "The direct causes (and effects) of events are near by, and even the indirect causes (and effects) are no further away than permitted by the velocity of light."[7] Then, citing a figure which is reproduced here in Fig. 1, Bell continues:

> "Thus for events in a space-time region 1… we would look for causes in the backward light cone, and for effects in the future light cone. In a region like 2, space-like separated from 1, we would seek neither causes nor effects of events in 1."[7]

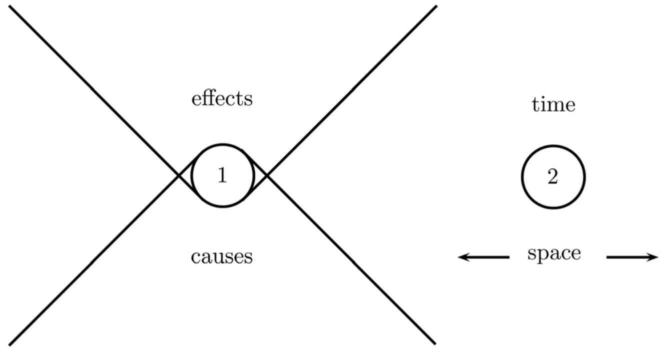

FIG. 1. Space-time location of causes and effects of events in region 1.

This formulation should be uncontroversial. Bell noted, however, that "[t]he above principle of local causality is not yet sufficiently sharp and clean for mathematics."[7]

Here is Bell's sharpened formulation. (The reader should understand that this formulation is, at this point, a "teaser" which those to whom it is not familiar should expect to understand only after further reading.)

> "A theory will be said to be locally causal if the probabilities attached to values of local beables in a space-time region 1 are unaltered by specification of values of local beables in a space-like separated region 2, when what happens in the backward light cone of 1 is already sufficiently specified, for example by a full specification of local beables in a space-time region 3…"[7]

The space-time regions referred to are illustrated in Fig. 2. We can express Bell's formulation mathematically as

$$P(b_1|B_3, b_2) = P(b_1|B_3), \tag{1}$$

where b_i refers to the value of a particular beable in space-time region i and B_i refers to a sufficient (for example, a complete) specification of all beables in the relevant region. (See Sec. III for the meaning of "beable.") P is the probability assigned to event b_1 by the theory in question, conditioned on the information specified after the vertical bar. Equation (1) captures just what Bell states in the caption of his accompanying figure (see Fig. 2): "full specification of [beables] in 3 makes events in 2 irrelevant for predictions about 1 in a locally causal theory."[7]

III. BEABLES

The first question about the word "beable" is: how to pronounce it? The word does not rhyme with "feeble," but with "agreeable." Bell invented the word as a contrast to the "observables" which play a fundamental role in the formulation of orthodox quantum theory.

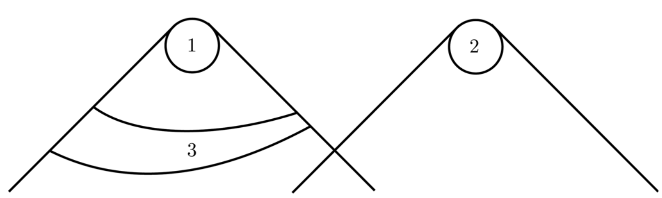

FIG. 2. Full specification of what happens in 3 makes events in 2 irrelevant for predictions about 1 in a locally causal theory.

A. Beables versus observables

Beables are those elements of a theory that are supposed to correspond to something that is physically real, independent of any observation: "The beables of the theory are those elements which might correspond to elements of reality, to things which exist. Their existence does not depend on 'observation.' Indeed observation and observers must be made out of beables."[35] As Bell explained,

> "The concept of 'observable'… is a rather woolly concept. It is not easy to identify precisely which physical processes are to be given the status of 'observations' and which are to be relegated to the limbo between one observation and another. So it could be hoped that some increase in precision might be possible by concentration on the *be*ables… because they are there."[36]

Bell's reservations about the concept of observable appearing in the formulation of a fundamental theory are closely related to the "measurement problem" of orthodox quantum mechanics, which Bell encapsulated by remarking that the orthodox theory is "unprofessionally vague and ambiguous"[35] in so far as its fundamental dynamics is expressed in terms of "words which, however legitimate and necessary in application, have no place in a *formulation* with any pretension to physical precision."[37] As Bell elaborated,

> "The concepts 'system,' 'apparatus,' 'environment,' immediately imply an artificial division of the world, and an intention to neglect, or take only schematic account of, the interaction across the split. The notions of 'microscopic' and 'macroscopic' defy precise definition. So also do the notions of 'reversible' and 'irreversible.' Einstein said that it is theory which decides what is 'observable.' I think he was right—'observation' is a complicated and theory-laden business. Then the notion should not appear in the *formulation* of fundamental theory."[37]

As Bell pointed out, even Bohr (a convenient personification of skepticism regarding the physical reality of unobservable microscopic phenomena) recognized certain objects (for example, the directly perceivable states of a classical measuring apparatus) as unambiguously real, that is, as beables:

> "The terminology, *be*-able as against *observ*-able, is not designed to frighten with metaphysic those dedicated to realphysic. It is chosen rather to help in making explicit some notions already implicit in, and basic to, ordinary quantum theory. For, in the words of Bohr, 'it is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms.' It is the ambition of the theory of local beables to bring these 'classical terms' into the equations, and not relegate them entirely to the surrounding talk."[36]

The vagueness and ambiguity of orthodox quantum theory is related to the fact that its formulation presupposes these classical, macroscopic beables, but fails to provide clear laws to describe them. As Bell explained,

> "The kinematics of the world, in [the] orthodox picture, is given by a wavefunction… for the quantum part, and classical variables—variables which *have* values—for the classical part [with the classical variables being] somehow macroscopic. This is not spelled out very explicitly. The dynamics is not very precisely formulated either. It includes a Schrödinger equation for the quantum part, and some sort of classical mechanics for the classical part, and 'collapse' recipes for their interaction."[37]

There are thus two related problems. First, the world as described by the theory is different on the two sides of what Bell called "the shifty split"[37]—that is, the division between "the quantum part" and "the classical part." The nature of the world posited by the theory thus remains vague as long as the dividing line between the macroscopic and microscopic remains undefined. Also the interaction across the split is problematic. Not only is the account of this dynamics (the "collapse" process) inherently bound up in concepts from Bell's list of dubious terms, but also the existence of a special dynamics for the interaction seems to imply inconsistencies with the dynamics already posited for the two realms separately. As Bell summarized,

> "I think there are *professional* problems [with quantum mechanics]. That is to say, I'm a professional theoretical physicist and I would like to make a clean theory. And when I look at quantum mechanics I see that it's a dirty theory. The formulations of quantum mechanics that you find in the books involve dividing the world into an observer and an observed, and you are not told where that division comes… So you have a theory which is fundamentally ambiguous…"[19]

This discussion should clarify the sort of theory Bell had in mind as satisfying the relevant standards of professionalism. It is often thought by those who do not understand or do not accept Bell's criticisms of orthodox quantum theory, that the concept of "beable," in terms of which his concept of local causality is formulated, commits one to hidden variables or determinism or some sort of naive realism or some other physically or philosophically dubious principle. But this is not correct. The requirement is only that fundamental theories, those "with any pretension to physical precision,"[37] be formulated clearly and precisely. According to Bell, such clarity and precision requires that the theories provide a uniform and consistent candidate description of physical reality. In particular, there should be no ambiguity or inconsistency regarding what a theory is fundamentally about (the beables), nor regarding how those posited physically real elements are assumed to act and interact (the laws).

B. Beables versus conventions

So far, we have explained the term "beable" by contrasting it to the "observables" of orthodox quantum theory. We must now also contrast the concept of "beables" with those elements of a theory which are conventional:

> "The word 'beable' will also be used here to carry another distinction, that familiar already in classical theory between 'physical' and 'non-physical' quantities. In Maxwell's electromagnetic theory, for example, the fields **E** and **H** are 'physical' (beables, we will say) but the potentials **A** and φ are 'non-physical.' Because of gauge invariance

> the same physical situation can be described by very different potentials. It does not matter [that is, it is not a violation of local causality] that in Coulomb gauge the scalar potential propagates with infinite velocity. It is not really supposed to *be* there. It is just a mathematical convenience."[36]

Or, as Bell explained it in another paper,

> "… there *are* things which *do* go faster than light. British sovereignty is the classical example. When the Queen dies in London (long may it be delayed) the Prince of Wales, lecturing on modern architecture in Australia, becomes *instantaneously* King… And there are things like that in physics. In Maxwell's theory, the electric and magnetic fields in free space satisfy the wave equation
>
> $$\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0$$
> $$\frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0$$
>
> … corresponding to propagation with velocity c. But the scalar potential, if one chooses to work in the Coulomb gauge, satisfies Laplace's equation
>
> $$-\nabla^2 \phi = 0$$
>
> … corresponding to propagation with infinite velocity. Because the potentials are only mathematical conveniences, and arbitrary to a high degree, made definite only by the imposition of one convention or another, this infinitely fast propagation of the Coulomb-gauge scalar potential disturbs no one. Conventions can propagate as fast as may be convenient. But then we must distinguish in our theory between what is convention and what is not."[7]

Consequently, to decide whether a given theory is or is not consistent with local causality,

> "you must identify in your theory 'local *be*ables.' The *be*ables of the theory are those entities in it which are, at least tentatively, to be taken seriously, as corresponding to something real. The concept of 'reality' is now an embarrassing one for many physicists…. But if you are unable to give some special status to things like electric and magnetic fields (in classical electromagnetism), as compared with the vector and scalar potentials, and British sovereignty, then we cannot begin a serious discussion."[7]

This explains why, for Bell, "It is in terms of local beables that we can hope to formulate some notion of local causality."[36]

C. Beables and candidate theories

It is important to appreciate that a beable is only a beable relative to some particular candidate theory which posits those elements as physically real (and gives precise laws for their dynamics). For example, the fields **E** and **B** (and not the potentials) are beables according to classical Maxwellian electrodynamics as it is usually understood. But, we could imagine an alternative theory which (perhaps motivated by the Aharanov-Bohm effect) posits the Coulomb gauge potentials as beables instead. Although this alternative theory would be empirically and mathematically equivalent to the usual theory, they would not have the same status regarding local causality. Where the usual Maxwellian theory respects local causality, the alternative theory would violate it: wiggling a charge would instantaneously affect the physically-real scalar potential at distant locations.

Thinking in terms of such candidate theories helps us to separate questions about what the "real beables" are—what really exists out there in physical reality—into two parts: what elements does a candidate theory posit as beables, and which candidate theory do we think is true? The point is that you do not have to be able to answer the second part to answer the first. This should provide some comfort to those who think we cannot establish a theoretical picture of external reality as true. Such people may still accept Bell's characterization of when "a theory will be said to be locally causal."[7]

But even those who are not skeptical on principle recognize that, because of the complexity in practice of settling questions about the truth status of scientific theories, some tentativeness is often in order. Bell recognizes this too:

> "I use the term 'beable' rather than some more committed term like 'being' or 'beer' to recall the essentially tentative nature of any physical theory. Such a theory is at best a *candidate* for the description of nature. Terms like 'being', 'beer', 'existent', etc., would seem to me lacking in humility. In fact 'beable' is short for 'maybe-able'."[35]

The crucial point is that "maybe" pertains to the epistemological status of a given candidate theory. In contrast, the "beable status"[36] of certain elements of a theory should be straightforward and uncontroversial. If there is any question about what elements a theory posits as beables, it can only be because the theory has not (yet) been presented in a sufficiently clear way. Whether the theory is true or false is a different question.

Bell did, however, take certain elements largely for granted as beables that any serious candidate theory would have to recognize as such: "The beables must include the settings of switches and knobs on experimental equipment, the currents in coils, and the readings of instruments."[36] As noted, even Bohr acknowledged the real existence (the beable status) of these sorts of things. And as suggested by Bohr, because our primary cognitive access to the world is through "switches and knobs on experimental equipment" and other such directly observable facts, it is difficult to imagine how one might take seriously a theory which didn't grant such facts beable status.[38]

We stress this point for two related reasons. First, anyone who is uncomfortable with the "metaphysical" positing of ultimate "elements of reality" should be relieved to find that the concept "beable" is merely a placeholder for whatever entities we tentatively include in the class which already, by necessity, exists and includes certain basic, directly perceivable features of the world around us. And second, these particular beables, for example, the settings of knobs and the positions of pointers, have a particularly central role to play in the derivation (see Sec. VI) of the empirically testable Bell inequalities.

IV. COMPLETENESS

We now turn to the last phrase in Bell's formulation of local causality:

> "A theory will be said to be locally causal if the probabilities attached to values of local beables in a space-time region 1 are unaltered by specification of values of local beables in a space-like separated region 2, when what happens in the backward light cone of 1 is already sufficiently specified, for example by a full specification of local beables in a space-time region 3…"[7]

The key assumption here is "that events in 3 be specified *completely*"[7] (emphasis added).

Let us first see why this requirement is necessary. Suppose that $\bar{B}_3$ denotes an *incomplete* specification of beables in region 3 (see Fig. 2). It can be seen that a violation of

$$P(b_1|\bar{B}_3, b_2) = P(b_1|\bar{B}_3) \tag{2}$$

does not entail the existence of any super-luminal causal influences. Suppose an event X in the overlapping backward light cones of regions 1 and 2 causally influences both b_1 and b_2. It might then be possible to infer from b_2, something about X, from which we could in turn infer something about b_1. Suppose, though, that the incomplete description of events in region 3, $\bar{B}_3$, omits precisely the "traces" of this past common cause X. Then, b_2 *could* usefully supplement $\bar{B}_3$ for predictions about 1; that is, Eq. (2) could be violated even in the presence of purely local causation.

Thus, as Bell explained, for Eq. (1) to function as a valid locality criterion,

> "it is important that events in 3 be specified completely. Otherwise the traces in region 2 of causes of events in 1 could well supplement whatever else was being used for calculating probabilities about 1. The hypothesis is that any such information about 2 becomes redundant when 3 is specified completely."[7]

Or as Bell explained in an earlier paper:

> "Now my intuitive notion of local causality is that events in 2 should not be 'causes' of events in 1, and vice versa. But this does not mean that the two sets of events should be uncorrelated, for they could have common causes in the overlap of their backward light cones [in a local theory]. It is perfectly intelligible then that if [B_3] in [region 3] does not contain a complete record of events in that [region], it can be usefully supplemented by information from region 2. So in general it is expected that [$P(b_1|b_2, \bar{B}_3) \neq P(b_1|\bar{B}_3)$.] However, in the particular case that [B_3] contains already a *complete* specification of beables in [region 3], supplementary information from region 2 could reasonably be expected to be redundant."[36]

Like the concept of beables itself, the idea of a sufficient (full or complete) specification of beables is relative to a given candidate theory. Bell's local causality condition requires that, to assess the consistency between a given theory and the relativistic causal structure sketched in Fig. 1, we must include in B_3 everything that the theory says is present (or relevant) in region 3. It is not necessary that we achieve omniscience regarding what actually exists in some spacetime region.

The appearance of the word "completeness" often reminds people of the EPR argument and suggests to some that Bell smuggled into his definition of local causality the unwarranted assumption that orthodox quantum theory is incomplete (see, for example, Ref. 39). As mentioned, Bell did accept the validity of the EPR argument. But this acceptance means only that, according to Bell, local causality (together with some of quantum mechanics' empirical predictions) entails the incompleteness of orthodox quantum mechanics. His view on this point is, however, no part of his *formulation* of local causality.

Although it is simplest to understand Bell's local causality condition as requiring a complete specification of beables in some spacetime region, there is an important reason why Bell explicitly left open the possibility that "what happens in the backward light cone of 1" might be "sufficiently specified" by something less than a complete specification of the beables there. This has to do with the fact (see Sec. VI) that to derive an empirically testable Bell-type inequality from the local causality condition, a subsidiary assumption is needed, sometimes called "experimental freedom" or "no conspiracies." This is in essence the assumption that, in the usual EPR-Bell kind of scenario in which a central source emits pairs of specially prepared particles in opposite directions toward two spatially separated measuring devices, it is possible for certain settings on the devices (determining which of several possible measurements are made on a given incoming particle) to be made "freely" or "randomly"—that is, independently of the state of the incident particle pair.

In more recent versions of these experiments, the relevant settings are made using independent (quantum) random number generators.[17] According to orthodox quantum mechanics, the outputs of such devices are genuinely, irreducibly random. Thus, for orthodox quantum mechanics, there is nothing in the past light cone of an individual measurement event foretelling which of the possible measurements will be performed. But there exist alternative candidate theories (such as the de Broglie-Bohm pilot-wave theory, which is deterministic) according to which those same settings *are* influenced by events in their past. But then, the relevant pasts of the device settings necessarily overlap with the pre-measurement states of the particles being measured. A complete specification of beables in the relevant region containing those pre-measurement states will therefore inevitably include facts relevant to (if not determining) the device settings. And so, in deriving the Bell inequality from local causality, there is a subtle tension between the requirement "that events in 3 be specified completely"[7] and the requirement that device settings can be made independent of the states of the particles-to-be-measured.

To resolve the tension, we need merely allow that the beables in the relevant region can be divided up into disjoint classes: those that are influenced by the preparation procedure at the source (and which thus encode the "state of the particle pair") and those that are to be used in the setting of the measurement apparatus parameters. Note that these two classes are likely to be far from jointly exhaustive: in any plausible candidate theory, there will have to exist many additional beables (corresponding, for example, to stray electromagnetic fields and low energy relic neutrinos) which are in neither of these classes. We thus expect a considerable "causal distance" between the two classes of beables (at least in a well-designed experiment). Such distance makes the "freedom" or "no conspiracies" assumption—namely, the absence of correlations between the two classes of beables—reasonable to accept.

This issue will be addressed in some more detail in Sec. VI. For now, we acknowledge its existence as a way of explaining why Bell's formulation of local causality mentions "complete" descriptions of events in region 3 as merely an example of the kind of description which is "sufficient." We summarize the discussion here as follows: what is required for the validity of the local causality condition is a complete specification of beables in region 3—but only those beables that are relevant in some appropriate sense to the event b_1 in question in region 1.

V. CAUSALITY

Recall the transition from Bell's preliminary, qualitative formulation of local causality to the final version. And recall, in particular, Bell's statement that the preliminary version was insufficiently sharp and clean for mathematics. What did Bell consider inadequate about the qualitative statement? It seems likely that it was the presence of the terms "cause" and "effect," which are notoriously difficult to define mathematically. About his final formulation Bell wrote: "Note, by the way, that our definition of locally causal theories, although motivated by talk of 'cause' and 'effect,' does not in the end explicitly involve these rather vague notions."[7]

How does Bell's "definition of locally causal theories" fail to "explicitly involve" the "rather vague notions" of cause and effect? On its face, this sounds paradoxical. But the resolution is simple: what Bell's definition actually avoids is any specific commitment about what physically exists and how it acts. (Any such commitments would seriously restrict the generality of the locality criterion, and hence undermine the scope of Bell's theorem.) Instead, Bell's definition shifts the burden of providing some definite account of causal processes to candidate theories and merely defines a space-time constraint that must be met if the causal processes posited by a candidate theory are to be deemed locally causal in the sense of special relativity.

The important mediating role of candidate theories regarding causality will be further stressed and clarified in Sec. V A. We then further clarify the concept of "causality" in Bell's "local causality" by contrasting it with several other ideas with which it has often been confused or conflated.

A. Causality and candidate theories

As discussed, according to Bell it is the job of physical theories to posit certain physically real structures (beables) and laws governing their evolution and interactions. Thus, Bell's definition of locally causal theories is not a specification of locality for a particular type of theory, namely, those that are "causal"—with the implication that there would exist also theories that are "non-causal." A theory, by the very nature of what we mean by that term in this context, is automatically causal. "Causal theory" is a redundancy. And so, as noted, we must understand Bell's "definition of locally causal theories" as a criterion that theories, that is, candidate descriptions of causal processes in nature, must satisfy to be in accord with special relativistic locality. As Bell explained, the practical reason for defining local causality in terms of the physical processes posited by some candidate theory (in contrast to the physical processes that actually exist in nature) has to do with our relatively direct access to the one as opposed to the other:

> "I would insist here on the distinction between analyzing various physical theories, on the one hand, and philosophising about the unique real world on the other hand. In this matter of causality it is a great inconvenience that the real world is given to us once only. We cannot know what would have happened if something had been different. We cannot repeat an experiment changing just one variable; the hands of the clock will have moved, and the moons of Jupiter. Physical theories are more amenable in this respect. We can *calculate* the consequences of changing free elements in a theory, be they only initial conditions, and so can explore the causal structure of the theory. I insist that [my formulation of the local causality concept] is primarily an analysis of certain kinds of physical theory."[40]

Bell's view, contrary to several commentators,[41,42] is that no special philosophical account of causation is needed to warrant the conclusion that violation of the locality condition implies genuine non-local causation. For Bell, it is a trivial matter to decide, for some (unambiguously formulated) candidate physical theory, what is and is not a causal influence. We can simply "explore the causal structure of" the candidate theory. This raises the question of how we might go from recognizing the non-locality of some particular candidate theory to the claim that *nature* is non-local. But that is precisely Bell's theorem: *all* candidate theories which respect the locality condition are inconsistent with experiment (see Sec. VI). Therefore, the "one true theory" (whatever that turns out to be) and hence nature itself must violate relativistic local causality.

B. Causality versus determinism

Section V A stressed that the "causal" in "locally causal theories" simply refers to the physically real existents and processes (beables and associated laws) posited by some candidate theory, whatever those might be. We in no way restrict the class of theories (whose locality can be assessed by Bell's criterion) by introducing "causality." In particular, the word "causal" in "locally causal theories" is not meant to imply or require that theories be deterministic in contrast to irreducibly stochastic:

> "We would like to form some [notion] of *local causality* in theories which are not deterministic, in which the correlations prescribed by the theory, for the beables, are weaker."[36]

Bell thus deliberately used the word "causal" as a wider abstraction that subsumes but does not necessarily entail determinism. This use is manifested most clearly in the fact that Bell's mathematical formulation of "local causality," Eq. (1), is stated in terms of probabilities. In Ref. 36, Bell discussed "local determinism" *first*, arguing that, in a "local deterministic" theory, the actual values of beables in region 1 (of Fig. 2) will be determined by a complete specification of beables in region 3 (with additional specification of beables from region 2 being redundant). In our notation, local determinism means

$$b_1(B_3, b_2) = b_1(B_3), \tag{3}$$

where b_1 and b_2 are the values of specific beables in regions 1 and 2, and B_3 denotes a sufficient (for example, complete) specification of beables in region 3.

In a (local) stochastic theory, even a complete specification of relevant beables in the past (for example, those in region 3 of Fig. 2) might not determine the realized value of the beable in question in region 1. Rather, the theory specifies only probabilities for the various possible values that might be realized for that beable. Note that determinism is not an alternative to but is rather a special case of stochasticity:

> "Consider for example Maxwell's equations, in the source-free case for simplicity. The fields **E** and **B** in region 1 are completely determined by the fields in region 3, regardless of those in 2. Thus this is a locally causal theory in the present sense. The deterministic case is a limit of the probabilistic case, the probabilities becoming delta functions."[7]

The natural generalization of our mathematical formulation of "local determinism" is Bell's local causality condition:

$$P(b_1|B_3, b_2) = P(b_1|B_3). \tag{4}$$

That is, b_2 is irrelevant—not for determining what happens in region 1 because that, in a stochastic theory, is not determined—but rather for determining the probability for possible occurrences in region 1. Such probabilities are the output of stochastic theories in the same sense that the actual realized values of beables are the output of deterministic theories. Thus, Bell's local causality condition for stochastic theories, Eq. (4), and the analogous condition, Eq. (3), for deterministic theories, impose the same locality requirement on the two kinds of theories: information about region 2 is irrelevant in regard to what the theory says about region 1, once the beables in region 3 are sufficiently specified.

If we insist that any stochastic theory is a stand-in for some (perhaps unknown) underlying deterministic theory (with the probabilities in the stochastic theory resulting not from indeterminism in nature, but from our ignorance), Bell's locality concept would cease to work. The requirement of a complete specification of beables in region 3 would contradict the allowance that such a specification does not necessarily determine the events in region 1. But this is no objection to Bell's concept of local causality. Bell did not ask us to accept that any particular theory (stochastic or otherwise) is true. Instead he just asked us to accept his definition of what it would mean for a stochastic theory to respect relativity's prohibition on superluminal causation. And this requires us to accept, at least in principle, that there could be an irreducibly stochastic theory and that the way causality appears in such a theory is that certain beables do, and others do not, influence the probabilities for specific events.

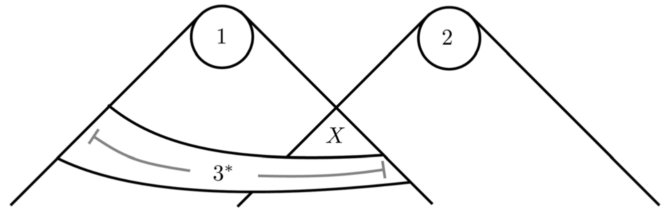

FIG. 3. Similar to Fig. 2, except that region 3^* (unlike region 3 of Fig. 2) fails to shield off region 1 from the overlapping backward light cones of regions 1 and 2. Thus, following the caption of Fig. 2, even full specification of what happens in 3^* does not necessarily make events in 2 irrelevant for predictions about 1 in a locally causal theory.

We stress here that the meaning of causality is broader than, and does not necessarily entail, determinism. Bell carefully formulated a local causality criterion that does not tacitly assume determinism, and which is stated explicitly in terms of probabilities—the fundamental, dynamical probabilities assigned by stochastic theories to particular events in space-time. The probabilities in Eq. (1) are not subjective in the sense of denoting the degree of someone's belief in a proposition about b_1; they cannot be understood as reflecting partial ignorance about the relevant beables in region 3; and they do not represent empirical frequencies for the appearance of certain values of b_1. They are, rather, the fundamental output of some candidate (stochastic) physical theory.

C. Causality versus correlation

Correlation doesn't imply causality. Two events (say, the values taken by beables b_1 and b_2 in Bell's spacetime regions 1 and 2, respectively) may be correlated without there necessarily being any implication that b_1 is the cause of b_2 or vice versa: "Of course, mere *correlation* between distant events does not by itself imply action at a distance, but only correlation between the signals reaching the two places."[13] Bell described the issue motivating his 1990 paper as "the problem of formulating… sharply in contemporary physical theory" "these notions, of cause and effect on the one hand, and of correlation on the other."[7]

It is sometimes reported that Bell's local causality condition is really only a "no correlation" requirement, such that the empirical violation of the resulting inequalities establishes only "non-local correlations" (rather than non-local causation) (see, for example, Ref. 29). But this is a misconception. Bell used the term "causality" (for example, in his "definition of locally causal theories") to highlight that a violation of this condition by some theory means that the theory posits non-local causal influences, rather than mere "non-local correlations."

It is helpful to illustrate this point by relaxing a requirement that Bell carefully incorporated into his formulation of local causality and showing that violation of the resulting weakened condition may still entail correlations between space-like separated events, but no longer implies that there are faster-than-light causal influences. We have done this once already in Sec. IV, where we explained why a violation of Eq. (2) would not—unlike a violation of Eq. (1)—entail a violation of the causal structure of Fig. 1. We now consider a second modified version of Bell's criterion.

Consider again the spacetime diagram in Fig. 2. Bell noted that "It is important that region 3 completely shields off from 1 the overlap of the backward light cones of 1 and 2."[7]

Why is this complete shielding so important? For example, why can we not replace region 3 of Fig. 2 with a region like that labeled 3^* in Fig. 3? This region, just like 3 in Fig. 2, closes off the back light cone of 1. So, it might seem like it would be sufficient for defining the probabilities associated with b_1 in a locally causal theory.

But a more careful analysis shows that a violation of

$$P(b_1|B_{3^*}, b_2) = P(b_1|B_{3^*}) \tag{5}$$

(the same as Eq. (1) but with region 3 of Fig. 2 replaced by region 3^* of Fig. 3) does not entail any non-local causation. Here, there is a perfectly local causal mechanism by which b_1 and b_2 can be correlated, in a way that isn't "screened off" by conditionalization on B_{3^*}, thus violating Eq. (5) in a situation that involves no violation of relativistic local causation. The mechanism is the following. In a stochastic theory, an event may occur at the space-time point labeled X in Fig. 3 which was not determined by the complete specification of beables B_{3^*} in region 3^*. But despite not having been determined by beables in its past, that event really comes into existence and may have effects throughout its future light cone, which includes both regions 1 and 2. Event X may broadcast sub-luminal influences which bring about correlations between b_1 and b_2, such that information about b_2 is not redundant in regard to defining what happens in region 1 (even after conditionalizing on B_{3^*}). Thus, we may have a violation of Eq. (5)—that is, a candidate theory could attribute different values to $P(b_1|B_{3^*}, b_2)$ and $P(b_1|B_{3^*})$—despite there being, according to the theory, no non-local causation at work. Although Eq. (5) may be described as a "no correlations" condition for regions 1 and 2, it definitely fails as a "no causality" condition.

If we return to the original region 3 of Fig. 2 which does "completely [shield] off from 1 the overlap of the backward light cones of 1 and 2,"[7] it is clear that no such correlation without non-local-causality can occur. Here, if some X-like event, not determined by even a complete specification of beables in region 3, occurs somewhere in the future light cone of region 3, it will necessarily fail to lie in the overlapping past light cones of regions 1 and 2, which would be necessary for it to in turn locally influence both of those events.

Bell carefully set things up so that a violation of Eq. (1) entails that there is some non-local causation. It is not necessarily that an event in region 2 causally influences events in region 1 or vice versa. It is possible, for example, that there is some other event, neither in region 1 nor region 2, which was not determined by B_3 and which causally influences both b_1 and b_2. The point is that this causal influence would have to be non-local; that is, it would have to violate the special relativistic causal structure in Fig. 1.[43]

To summarize the point that a violation of Eq. (1) entails non-local causation rather than mere correlations between space-like separated events, it is helpful to recall Bell's example of the correlation between the ringing of a kitchen alarm and the readiness of a boiling egg. That the alarm rings just as the egg is finished cooking does not entail or suggest that the ringing caused the egg to harden. Correlation does not imply causality. As Bell completes the point, "The ringing of the alarm establishes the readiness of the egg. But if it is already given that the egg was nearly boiled a second before, then the ringing of the alarm makes the readiness no more certain."[7]

If we replace b_2 for "the ringing of the alarm," b_1 for "the readiness of the egg," and B_3 for "the egg was nearly boiled a second before," we have a simple example of Eq. (1): although b_1 and b_2 may be correlated such that information about b_2 can tell us something about b_1, that information is redundant in a locally causal theory once B_3 is specified.

D. Causality versus signaling

An idea that is often confused with local causality is local (that is, exclusively slower-than-light) signaling.[44] Signaling is a human activity in which one person transmits information, across some distance, to another person. Such transmission requires a causal connection between the sending event and the receiving event and requires the ability of the two people to send and receive the information. That is, signaling requires some measure of control over appropriate beables on the part of the sender and some measure of access to appropriate beables on the part of the recipient.

The requirement that theories prohibit the possibility of faster-than-light signaling, which is all that is imposed in relativistic quantum field theory by the requirement that field operators at spacelike separation commute,[36] is a much weaker condition than the prohibition of faster-than-light causal influences. Theories can exhibit violations of relativistic local causality and yet, because certain beables are inadequately controllable by and/or inadequately accessible to humans, preclude faster-than-light signals. Orthodox quantum mechanics including ordinary relativistic quantum field theory is an example of such a theory. Another example is the pilot-wave theory of de Broglie and Bohm, in which "… the consequences of events at one place propagate to other places faster than light. This happens in a way that we cannot use for signaling. Nevertheless it is a gross violation of relativistic causality."[15] One of the most common mistakes made by commentators on Bell's theorem is to conflate local causality with local signaling.[45] Often this conflation takes the form of a double-standard in which alternatives to ordinary quantum mechanics are dismissed as non-local and therefore unacceptable on the grounds that they include (either manifestly, as in pilot-wave theory, or in principle, as established by Bell's theorem) "gross violations of relativistic causality." But ordinary quantum mechanics is argued by comparison to be perfectly local, where now only "local signaling" is meant. Such reasoning is clearly equivocal once

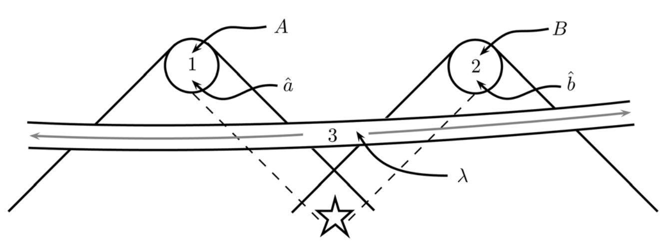

FIG. 4. Space-time diagram illustrating the various beables of relevance for the EPR-Bell setup (see Bell's diagram in Ref. 7). Separated observers Alice (in region 1) and Bob (in region 2) make spin-component measurements using apparatus settings $\hat{a}$ and $\hat{b}$, respectively, on a pair of spin- or polarization-entangled particles as indicated by the dashed lines. The measurements have outcomes A and B, respectively. The state of the particle pair in region 3 is denoted by λ. Note that region 3 extends across the past light cones of both regions 1 and 2. It thus not only "completely shields off from 1 the overlap of the backward light cones of 1 and 2"[7] but does so also for region 2. Bell's local causality condition therefore requires both that $\hat{b}$ and B are irrelevant for predictions about the outcome A, and that $\hat{a}$ and A are irrelevant for predictions about the outcome B, once λ is specified.

we appreciate that local causality and local signaling have different meanings.

Differentiating these two notions raises the question of what special relativity should be understood to prohibit. But the idea that the relativistic causal structure, sketched in Fig. 1, should somehow apply exclusively to the narrowly human activity of signaling, seems highly dubious:

> "… the 'no signaling…' notion rests on concepts which are desperately vague, or vaguely applicable. The assertion that 'we cannot signal faster than light' immediately provokes the question:
>
> Who do we think *we* are?
>
> *We* who can make 'measurements,' *we* who can manipulate 'external fields,' *we* who can 'signal' at all, even if not faster than light? Do *we* include chemists, or only physicsts, plants, or only animals, pocket calculators, or only mainframe computers?"[7]

That is, the idea that special relativity is compatible with non-local causal influences (but only prohibits non-local signaling) seems afflicted by the same problem that afflicts theories whose formulations involve words such as "observable," "microscopic," and "environment." In particular, the notion of signaling seems too superficial and too anthropocentric to adequately capture the causal structure of Fig. 1.

VI. IMPLICATIONS OF LOCAL CAUSALITY

Having reviewed Bell's careful formulation of relativistic local causality, let us now indicate some of its important consequences.

A. Factorization

A typical EPR-Bell setup involves separated observers (Alice and Bob) making spin-component measurements using Stern-Gerlach devices oriented spatially along the $\hat{a}$ and $\hat{b}$ directions, respectively, on each of a pair of spin-entangled particles. The outcomes of their individual measurements—manifested in the final location of the particle or the position of some pointer or some fact about some other beable—are denoted by A and B, respectively.

The beables relevant to a given run of the experiment may be cataloged as in Fig. 4. We may roughly think of $\hat{a}$ and $\hat{b}$, which are in regions 1 and 2, respectively, as referring to the spatial orientations of the two pieces of the measuring apparatus and λ in region 3 as referring to the state of the particle pair emitted by the source. (The phrase "state of the particle pair" should not be taken too seriously, because no actual assumption is made about the existence, for example, of literal particles.)

Unlike region 3 of Fig. 2, region 3 of Fig. 4 extends across the past light cone not only of region 1, but of region 2 as well. It particular, this extended region closes off the past light cones of regions 1 and 2 and shields both regions from their overlapping past light cones. A complete specification of beables in region 3 will thus, according to Bell's concept of local causality, "make events in 2 irrelevant for predictions about 1,"[7] and will also make events in 1 irrelevant for predictions about 2:

$$P(A|\hat{a},\hat{b},B,\lambda) = P(A|\hat{a},\lambda), \tag{6}$$

and

$$P(B|\hat{a},\hat{b},\lambda) = P(B|\hat{b},\lambda). \tag{7}$$

From Eqs. (6), (7), and the identity

$$P(A,B|\hat{a},\hat{b},\lambda) = P(A|\hat{a},\hat{b},B,\lambda)\cdot P(B|\hat{a},\hat{b},\lambda), \tag{8}$$

the factorization of the joint probability for outcomes A and B immediately follows:

$$P(A,B|\hat{a},\hat{b},\lambda) = P(A|\hat{a},\lambda)\cdot P(B|\hat{b},\lambda) \tag{9}$$

This factorization condition is widely recognized to be sufficient for the derivation of empirically testable Bell-type inequalities. As Bell notes, however, "Very often such factorizability is taken as the starting point of the analysis. Here, we have preferred to see it not as the *formulation* of 'local causality,' but as a consequence thereof."[7]

B. The EPR argument

In their famous paper, Einstein, Podolsky, and Rosen argued that a local explanation for the perfect correlations predicted by quantum theory required the existence of locally pre-determined values for the measurement outcomes.[9] Because ordinary quantum mechanics contains no such elements of reality, EPR concluded that ordinary quantum mechanics (and the wave function in particular) did not provide a complete description of physical reality. They suggested that an alternative, locally causal theory which provides a complete description of physical reality might be found.

If we assume that the relevant empirical predictions of quantum theory are correct, we can summarize the logic of EPR's argument as

$$\text{locality} \rightarrow \text{incompleteness}, \tag{10}$$

where 'incompleteness' means the incompleteness of the orthodox quantum mechanical description of the particles in question (in terms of their quantum state alone). This statement is logically equivalent to the statement that

$$\text{completeness} \rightarrow \text{non-locality} \tag{11}$$

which explains why the EPR argument is sometimes characterized as an argument for the incompleteness of orthodox quantum mechanics and sometimes as pointing out the non-locality of this theory.

In their argument, EPR appealed to an intuitive notion of local causality which was not precisely formulated; but the argument can be made rigorous by using Bell's formulation of local causality. It is clarifying to begin with the EPR argument in the form of statement (11). The proof consists in using the notion of local causality in its directly intended way, namely, to assess whether a particular candidate theory is or is not local.

Take again the situation indicated in Fig. 4. Because of the structure of region 3—note that it could be extended into a space-like hypersurface crossing through the region 3 depicted in Fig. 4 and still satisfy the requirements discussed earlier—the relevant complete specification of beables does not presuppose that the state λ of the particle pair must factorize into two distinct and independent states for the two particles. The state can instead be characterized in a way that is inseparable, as in ordinary quantum mechanics, and the argument still holds: "It is notable that in this argument nothing is said about the locality, or even localizability, of the variable λ. These variables could well include, for example, quantum mechanical state vectors, which have no particular localization in ordinary space-time. It is assumed only that the outputs A and B, and the particular inputs a and b, are well localized."[13]

Let us suppose that the preparation procedure at the particle source (the star in Fig. 4) gives rise to a particle pair in the spin singlet state as described by ordinary quantum mechanics

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2), \tag{12}$$

where $|\uparrow\rangle_1$ means that particle 1 is spin-up along the z-direction, etc. (The full quantum state of the particle pair will include also spatial degrees of freedom. These will not enter into the argument, though, and so are suppressed for simplicity.)

Suppose for example that $\hat{a} = \hat{b} = \hat{z}$, that is, both Alice and Bob choose to measure the spins of the incoming particles along the z-direction. Then quantum mechanics predicts (letting $A = +1$ denote that Alice finds her particle to be spin up, etc.) that either $A = +1$ and $B = -1$ (with probability 50%) or $A = -1$ and $B = +1$ (with probability 50%).

For orthodox quantum mechanics λ in Fig. 4 is just the quantum state of Eq. (12), and we have, for example, that

$$P(A = +1|\hat{a}, \lambda) = \frac{1}{2} \tag{13}$$

while

$$P(A = +1|\hat{a}, \hat{b}, \lambda, B = -1) = 1, \tag{14}$$

in violation of Eq. (1). As Bell explains

> "The theory requires a perfect correlation of [results] on the two sides. So specification of the result on one side permits a 100% confident prediction of the previously totally uncertain result on the other side. Now in ordinary quantum mechanics there just *is* nothing but the wavefunction for calculating probabilities. There is then no question of making the result on one side redundant on the other by more fully specifying events in some space-time region 3. We have a violation of local causality."[7]

As mentioned, Statements (10) and (11) are logically equivalent, so a locally causal explanation for the perfect correlations predicted by quantum mechanics requires a theory with more (or different) beables than just the wave function. It is possible to show directly from Bell's concept of local causality that we must, in particular, posit beables which pre-determine the outcomes of both measurements.

To begin, we drop the assumption, which holds for ordinary quantum mechanics, that it is possible to fully control the state λ produced by the preparation procedure at the source. Instead, we allow that λ may take several distinct values from one run of the experiment to another. We also assume, for simplicity, that Alice and Bob freely choose to make measurements along the $\hat{z}$ direction.

The argument is then simple: we have already shown that local causality entails the factorization of the joint probability for outcomes A and B once λ is specified. Consider a joint event, such as $A = +1$, $B = +1$, whose joint probability vanishes. Factorization then implies that, for each value of λ that might with nonzero probability be produced by the preparation procedure, either $P(A = +1|\hat{a}, \lambda)$ or $P(B = +1|\hat{b}, \lambda)$ must vanish.

But because there are only two possible outcomes for each measurement, each of these possibilities entails that the opposite outcome is pre-determined. For example,

$$P(A = +1|\hat{a}, \lambda) = 0 \quad \rightarrow \quad P(A = -1|\hat{a}, \lambda) = 1, \tag{15}$$

which means that those values of λ to which this statement applies must encode the outcome $A = -1$, which will then be revealed with certainty if a measurement along $\hat{a}$ is performed. The possible values of λ must therefore fall into two mutually exclusive and jointly exhaustive categories—those that encode the pre-determined outcomes $A = +1$ and $B = -1$ and those that encode the pre-determined outcomes $A = -1$ and $B = +1$.[46]

Furthermore, because the measurement axes are assumed to be free, the same argument can establish that λ must encode pre-determined outcomes for *all* possible measurement directions. We thus see how theories of deterministic hidden variables (or what Mermin has dubbed "instruction sets"[47]) are required, by local causality, to explain the perfect correlations predicted by ordinary quantum mechanics.

C. The Clauser-Horne-Shimony-Holt inequality

It is well known that a Bell-type inequality follows from the assumption of local deterministic hidden variables or "instruction sets." That theories of this type are actually required by locality (as explained in Sec. VI B) should therefore already explain the seriousness with which Bell took the idea of a fundamental conflict between relativistic locality and the predictions of quantum mechanics. This conflict can be brought out in an even more direct way by deriving a Bell-type inequality directly from the factorization of the joint probability as in Eq. (9)—and hence from Bell's local causality (without any additional discussion of determinism or pre-determined values).

Assume that the measurement scenario indicated in Fig. 4 is repeated many times, with each setting being selected randomly on each run from two possibilities: $\hat{a} \in \{\hat{a}_1, \hat{a}_2\}$, $\hat{b} \in \{\hat{b}_1, \hat{b}_2\}$. The procedure that prepares the particles to be measured is held fixed for all runs of the experiment. As before, this does not necessarily imply that λ is constant for all runs, because the relevant beables may be less than fully controllable. We will assume that the distribution of different values of λ for the runs can be characterized by a probability distribution $\rho(\lambda)$.

We define the correlation of (± 1-valued) outcomes A and B as the expected value of their product:

$$C(\hat{a}, \hat{b}) = \int \sum_{A,B} A\, B\, P\,(A|\hat{a}, \lambda) P(B|\hat{b}, \lambda)\rho(\lambda) d\lambda \tag{16a}$$

$$= \int \bar{A}(\hat{a}, \lambda)\bar{B}(\hat{a}, \lambda)\rho(\lambda) d\lambda, \tag{16b}$$

where $\bar{A}(\hat{a},\lambda) \equiv P(A=+1|\hat{a},\lambda) - P(A=-1|\hat{a},\lambda)$ satisfies $|\bar{A}| \leq 1$ and similarly for $\bar{B}$.

Now we consider several combinations of correlations involving different pairs of settings. To begin with,

$$C(\hat{a}_1,\hat{b}_1) \pm C(\hat{a}_1,\hat{b}_2) = \int \bar{A}(\hat{a}_1,\lambda)\left[\bar{B}(\hat{b}_1,\lambda) \pm \bar{B}(\hat{b}_2,\lambda)\right]\rho(\lambda)d\lambda \quad (17)$$

so that

$$\left|C(\hat{a}_1,\hat{b}_1) \pm C(\hat{a}_1,\hat{b}_2)\right| \leq \int \left|\bar{B}(\hat{b}_1,\lambda) \pm \bar{B}(\hat{b}_2,\lambda)\right|\rho(\lambda)d\lambda. \quad (18)$$

Similarly, we have that

$$\left|C(\hat{a}_2,\hat{b}_1) \mp C(\hat{a}_2,\hat{b}_2)\right| \leq \int \left|\bar{B}(\hat{b}_1,\lambda) \mp \bar{B}(\hat{b}_2,\lambda)\right|\rho(\lambda)d\lambda. \quad (19)$$

By adding Eqs. (18) and (19) and noting that $|x \pm y| + |x \mp y|$ equals $2x$, $-2x$, $2y$, or $-2y$, it follows immediately that

$$\left|C(\hat{a}_1,\hat{b}_1) \pm C(\hat{a}_1,\hat{b}_2)\right| + \left|C(\hat{a}_2,\hat{b}_1) \mp C(\hat{a}_2,\hat{b}_2)\right| \leq 2, \quad (20)$$

which is the Clauser-Horne-Shimony-Holt inequality.[39] This inequality is in essence the relation tested in experiments such as those discussed in Refs. 17 and 18. Quantum theory predicts that (for appropriate preparations of the two-particle state and for appropriate choices of $\hat{a}_1$, $\hat{a}_2$, $\hat{b}_1$, and $\hat{b}_2$) the left-hand side of Eq. (20) should be $2\sqrt{2}$, which is more than 40% larger than the constraint implied by local causality. The experimental results are in excellent agreement with the quantum predictions.

Because the inequality is derived from the local causality condition, it follows from the experimental results that any theory which makes empirically correct predictions will have to violate the local causality condition. As Bell wrote, "The obvious definition of 'local causality' does not work in quantum mechanics, and this cannot be attributed to the 'incompleteness' of that theory."[7]

D. The "free variables" assumption

We return to the assumption that the settings $\hat{a}$ and $\hat{b}$ are random or free. In terms of the derivation we have just presented, the assumption is that the probability distribution $\rho(\lambda)$ for the distribution of possible states of the particle pair created by the source is independent of the apparatus settings $\hat{a}$ and $\hat{b}$. For example, in deriving Eq. (17), we assumed that the same probability distribution $\rho(\lambda)$ characterizes the runs in which $\hat{a}_1$ and $\hat{b}_1$ are measured, as characterizes the runs in which $\hat{a}_1$ and $\hat{b}_2$ are measured. As Bell explained,

> "we may imagine the experiment done on such a scale, with the two sides of the experiment separated by a distance of order light minutes, that we can imagine these settings being freely chosen at the last second by two different experimental physicists.... If these last second choices are truly free..., they are not influenced by the variables λ. Then the resultant values for $[\hat{a}]$ and $[\hat{b}]$ do not give any information about λ. So the probability distribution over λ does not depend on $[\hat{a}]$ or $[\hat{b}]$..."[7]

Of course, the real experiments do not involve "settings being freely chosen at the last second by two different experimental physicists," but instead involve physical random number generators. As mentioned, this means that at least in principle, for some possible candidate theories, a complete description of beables in region 3 of Fig. 4 includes not only a complete description of the state of the particle pair but also a complete description of whatever physical degrees of freedom will determine the eventual settings $\hat{a}$ and $\hat{b}$—making it not only possible but likely that the candidate theory should exhibit (contrary to the assumption that was made) correlations between what we have called λ and those settings.

As suggested earlier, though, we can appeal to the expectation that serious candidate theories will posit an enormously large number of physical degrees of freedom in a spacetime region such as 3, only some tiny fraction of which are actually needed to completely specify the state of the particle pair, that is, the beables that are physically influenced by the preparation procedure at the source. There are then still many other beables in region 3 which might be used to determine/influence the apparatus settings. The "free variables" assumption is that these settings are somehow made such that there are no correlations between the beables used to determine the apparatus settings and those that encode the state of the particle pair.

As Bell acknowledged, one logical possibility in the face of the empirical violations of the Clauser-Horne-Shimony-Holt inequality is that

> "it is not permissible to regard the experimental settings $[\hat{a}]$ and $[\hat{b}]$ in the analyzers as independent of the supplementary variables λ, in that $[\hat{a}]$ and $[\hat{b}]$ could be changed without changing the probability distribution $\rho(\lambda)$. Now even if we have arranged that $[\hat{a}]$ and $[\hat{b}]$ are generated by apparently random radioactive devices, housed in separated boxes and thickly shielded, or by Swiss national lottery machines, or by elaborate computer programmes, or by apparently free willed experimental physicists, or by some combination of all of these, we cannot be *sure* that $[\hat{a}]$ and $[\hat{b}]$ are not significantly influenced by the same factors λ that influence A and B. But this way of arranging quantum mechanical correlations would be even more mind boggling than one in which causal chains go faster than light. Apparently separate parts of the world would be deeply and conspiratorially entangled, and our apparent free will would be entangled with them."[13]

Bell introduced the term "superdeterministic" to describe theories which explain the empirically observed correlations by denying that the apparatus settings can be treated as free:

> "An essential element in the reasoning here is that $[\hat{a}]$ and $[\hat{b}]$ are free variables. One can envisage then theories in which there just *are* no free variables for the polarizer angles to be coupled to. In such 'superdeterministic' theories the apparent free will of experimenters, and any other apparent randomness, would be illusory. Perhaps such a theory could be both locally causal and in agreement with quantum mechanical predictions. However I do not expect to see a serious theory of this kind. I would expect a serious theory to permit 'deterministic chaos' or 'pseudorandomness,' for complicated subsystems

(for example computers) which would provide variables sufficiently free for the purpose at hand. But I do not have a theorem about that."[7]

It is sometimes erroneously thought that the "freedom" or "no conspiracies" assumption follows (or should follow) from local causality. For example, Shimony, Horne, and Clauser[48] criticized Bell's derivation for using (in our notation) the assumption $\rho(\lambda|\hat{a},\hat{b}) = \rho(\lambda)$ which, they correctly pointed out, does not follow from local causality. Bell subsequently clarified that it was a separate assumption, not supposed to follow from local causality. And, as articulated by the discussants in Ref. 48, the additional assumption seems eminently reasonable:

"… we feel that it is wrong on methodological grounds to worry seriously about [the possibility of the kind of conspiracy that would render the assumption inapplicable] if no specific causal linkage [between the beables λ and those which determine the apparatus settings] is proposed. In any scientific experiment in which two or more variables are supposed to be randomly selected, one can always conjecture that some factor in the overlap of the backward light cones has controlled the presumably random choices. But, we maintain, skepticism of this sort will essentially dismiss all results of scientific experimentation. Unless we proceed under the assumption that hidden conspiracies of this sort do not occur, we have abandoned in advance the whole enterprise of discovering the laws of nature by experimentation."[48]

Imagine, for example, an experimental drug trial in which patients are randomly selected to receive either the drug or a placebo. It is logically possible that the supposedly random selections (made, say, by flipping a coin) are correlated with some pre-existing facts about the health of the patients. Such a correlation could skew the results of the trial, resulting say in a statistically significant improvement in the health of the patients given the genuine drug even though the drug is impotent or worse. The suggestion is that unless there is some plausible causal mechanism that might conceivably produce the correlations in question, it is reasonable to assume that the conspiratorial correlations are absent. That is, the additional assumption beyond local causality which is needed to derive the Clauser-Horne-Shimony-Holt inequality "is no stronger than one needs for experimental reasoning generically."[48] The "no conspiracies" assumption thus falls into the same category as, for example, the validity of logic and certain mathematical operations, which, although used in the derivation, are not seriously challengeable. This explains why we sometimes do not even bother to mention this assumption as, for example, when writing that the Clauser-Horne-Shimony-Holt inequality follows from Bell's concept of local causality alone.

VII. SUMMARY AND OPEN QUESTIONS

We have reviewed Bell's formulation of relativistic local causality, including a survey of its conceptual background and a sketch of its most important implications. We have stressed that Bell's formulation does not presuppose determinism or the existence of hidden variables, but instead seems perfectly to capture the intuitive idea, widely taken as an implication of special relativity, that causal influences cannot propagate faster than light. And as we have seen, now taking the "no conspiracies" assumption for granted, the empirically violated Clauser-Horne-Shimony-Holt inequality can be derived from Bell's concept of local causality alone, without the need for further assumptions involving determinism, hidden variables, "realism," or anything of that sort.

This hopefully clarifies why Bell disagreed with the widespread opinion that his theorem and the associated experiments vindicate ordinary quantum theory as against hidden variable theories or vindicate Bohr's philosophy as against Einstein's. Instead, for Bell, "the real problem with quantum theory" is the "apparently essential conflict between any sharp formulation and relativity [that is, the] apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory…."[15]

Although we have argued strongly for the reasonableness of Bell's formulation of relativistic local causality, this particular formulation should not necessarily be regarded as definitive. We briefly indicate several points on which its applicability to various sorts of exotic theories could be questioned, and where a more general or distinct formulation of local causality might be sought. For example, we might worry that a theory with a non-Markovian character (that is, a theory in which causal influences can jump discontinuously from one time to a later time) could violate Bell's local causality condition despite positing no strictly faster-than-light influences. The idea would be that influences could "hop over" region 3 of Fig. 2, leading to correlations in regions 1 and 2 but leaving no trace in 3. This shortcoming in the formulation could seemingly be addressed by requiring that Bell's region 3 cover a region of spacetime so "thick" (in the temporal direction) that hopping non-Markovian influences could not make it across. In the limit of arbitrarily large violations of the Markov property, this change would require region 3 to encompass the entire past light cone of the region 3 in Fig. 2. But this fix would come at a price: the more of spacetime that is included in region 3, the more difficult it will be to argue for the reasonableness of the "no conspiracies" assumption, and the more we might worry that the condition could fail to detect certain kinds of non-localities such that it would function, no longer as a *formulation* of locality, but merely as a *necessary condition* for locality.

Similar problems arise when we contemplate the possibility of theories that posit not only local beables, that is, those "associated with definite positions in space"[35] but also non-local beables. The de Broglie-Bohm pilot-wave theory is probably the clearest example: its posited ontology includes both particles (which follow definite trajectories in 3-space and are pre-eminent examples of local beables) and a guiding wave (which is just the usual quantum mechanical wave function, interpreted as a beable). For an N-particle system, the wave function is a function on the $3N$-dimensional configuration space, so if it is a beable, it is a non-local beable.[49]

As mentioned, Bell's region 3 can be extended into a space-like hypersurface without spoiling any of the arguments that have been given in this paper. We may then include, as well, where Bell's formulation instructs us to use a complete specification of the local beables in region 3, values for any non-local beables which, like wave functions, can be associated with hypersurfaces. And it is important that, even when including information about non-local beables in this way, the local causality condition is violated by the pilot-wave theory. (Note that the argument in Sec. VI B for the non-local character of orthodox quantum mechanics was of just this type.) So again, for theories involving non-local beables, Bell's formulation can be easily tweaked to yield a necessary condition for local causality, which condition is unambiguously violated by various extant and obviously-nonlocal theories.

Still, as formulated, Bell's concept of local causality seems to presuppose that we are dealing with theories positing exclusively local beables.[50] It can be stretched to accommodate certain extant theories which also posit non-local beables, but how to formulate the concept with complete generality and what other issues (like those encountered for non-Markovian theories) may arise in the attempt, remains unclear. Of course, it is also unclear how seriously we should take theories with non-local beables in the first place. In particular, should such theories even be considered candidates for "locally causal" status? And could a theory positing non-local beables be genuinely consistent with special relativity?

Such questions will not be answered here. We raise them only to give readers some sense of the concerns that they might have about Bell's formulation of local causality. Their admittedly exotic character should help explain why Bell felt driven to contemplate "unspeakable" deviations from conventional wisdom. In particular, we can now appreciate how simple everything would become if we dropped the insistence on reconciling the Bell experiments with "fundamental relativity" and instead returned to the pre-Einstein view according to which there exists a preferred frame of reference. As explained by Bell, such a view can accommodate faster-than-light causal influences much more easily than the usual Einsteinian understanding of relativity.

Our goal here, though, is not to lobby for this view, but merely to explain Bell's rationale for taking it seriously as a possibility warranting attention, not just by philosophers, but by physicists interested in addressing the puzzles of yesterday, today, and tomorrow.

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[4]Isaac Newton, February 25, 1693 letter to Richard Bentley. See, for example, Andrew Janiak, "Newton's philosophy," in *The Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta, <plato.stanford.edu/archives/win2009/entries/newton-philosophy/>.

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[9]Albert Einstein, Boris Podolsky, and Nathan Rosen, "Can quantum-mechanical description of reality be considered complete?," Phys. Rev. **47**, 777–780 (1935).

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[11]The terminology of "hidden variables" is unfortunate because, at least in the one existing example of a serious hidden variables theory (the de Broglie-Bohm "pilot wave" theory, which adds to the standard quantum mechanical wave function definite particle positions obeying a deterministic evolution law), the "hidden variables" are not hidden. In Ref. 18, Bell remarked that "it would be appropriate to refer to the $\times$s as 'exposed variables' and to ψ as a 'hidden variable.' It is ironic that the traditional terminology is the reverse of this." Similarly in Ref. 12, he writes "Although [in Bohmian mechanics] Ψ is a real field it does not show up immediately in the result of a single 'measurement,' but only in the statistics of many such results. It is the de Broglie-Bohm variable X that shows up immediately each time. That X rather than Ψ is historically called a 'hidden' variable is a piece of historical silliness."12 It is also relevant that the wave function ψ is "hidden" in the sense of being not accessible via experiment, even in orthodox quantum theory, which is the primary example of a non-hidden-variable theory. See Roderich Tumulka, "Understanding Bohmian mechanics: A dialogue," Am. J. Phys. **72**(9), 1220–1226 (2004).

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[31]John S. Townsend, *A Modern Approach to Quantum Mechanics* (McGraw-Hill, New York, 1992).

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[33]For Bell, there was no important distinction between "locality" and "local causality." For example, Bell first used the phrase "local causality" in print in Ref. 36. In the same paper, he refers to the inequality (which he has shown how to derive from "local causality") as "the locality inequality" and remarks that the detailed discussion of "local causality" in Sec. 2 was "an attempt to be rather explicit and general about the notion of locality, along lines only hinted at in previous publications," see Ref. 36. This usage is consistent with his later publications. See, for example, Ref. 8, pp. xi–xii and Ref. 7.
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[44]What we call "local signaling" is also sometimes called "no signaling" or "signal locality."
[45]See, for example, Refs. 33 and 34 and Leslie E. Ballentine and Jon P. Jarrett, "Bell's theorem: Does quantum mechanics contradict relativity?"Am. J. Phys. **55**(8), 696–701 (1987).
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[49]About the de Broglie-Bohm theory, Bell wrote: "No one can understand this theory until he is willing to think of ψ as a real objective field rather than just a 'probability amplitude.' Even though it propagates not in 3-space but in 3N-space" in Ref. 18. Some proponents of the de Broglie-Bohm pilot wave theory prefer to interpret the wave function in that theory not as a beable, but rather as a law. See, for example, S. Goldstein and N. Zanghi, 1618 "Reality and the role of the wavefunction in quantum theory," forthcoming in D. Albert and A. Ney, eds. *The Wave Function: Essays in the Metaphysics of Quantum Mechanics* (Cambridge, UP., 2011).
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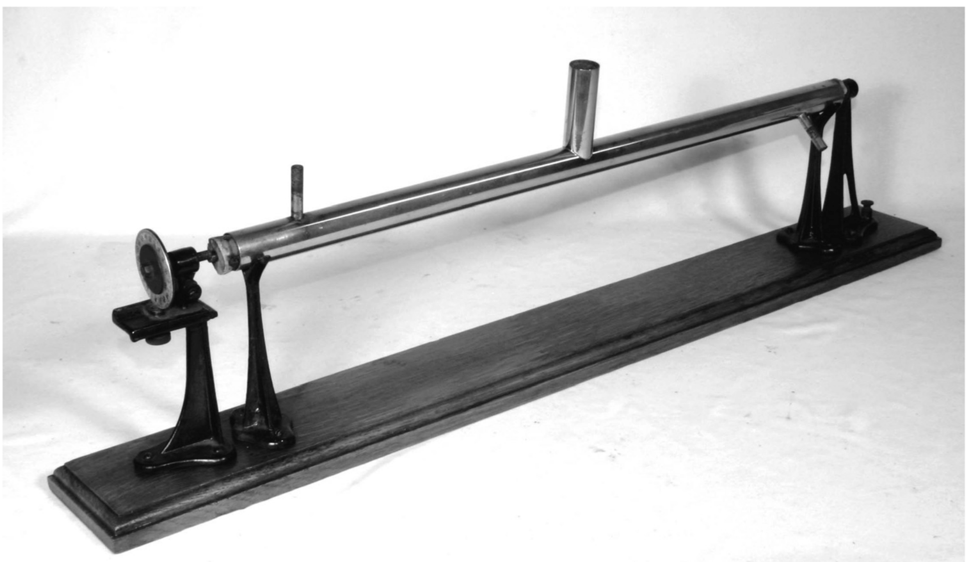

Knott Thermal Expansion Apparatus. This device allows quantitative measurements to be made of the coefficient of thermal expansion of a metallic rod. Steam or hot water is introduced into the metal jacket surrounding the rod, and the temperature noted with a thermometer inserted into the upright tube. The micrometer screw at the end is advanced until it contacts the rod, thus completing an electric circuit. This is repeated at a series of temperatures, permitting a graph of length as a function of temperature to be made. The apparatus is in the Greenslade Collection. (Notes and photograph by Thomas B. Greenslade, Jr., Kenyon College)