with $\epsilon_1^2 = |E^2 - \epsilon_0^2|$, $\epsilon_2^2 = [(E + \hbar\omega)^2 - \epsilon_0^2]$, $g(E) = [E^2 + \epsilon_0^2 + E\hbar\omega]/\epsilon_1\epsilon_2$, and $\alpha = R/\hbar v_0$, where ϵ_1 and ϵ_2 are the Bloch energies corresponding, respectively, to E and $E + \hbar\omega$, f is the Fermi–Dirac distribution function, and v_0 is the velocity at the Fermi surface.

For reference to Fig. 2, $\tilde{\sigma}/\sigma_n = (\sigma_1 - i\sigma_2)/\sigma_n$.

[1] T. Van Duzer and C. W. Turner, *Principles of Superconductive Devices and Circuits* (Elsevier, New York, 1981).

[2] The nonlocal theory was put on a firm theoretical footing by G. C. Reuter and E. H. Sondheimer, "The theory of the anomalous skin effect in metals," Proc. R. Soc. London Ser. A **195**, 336–364 (1948).

[3] Reference 1, Sec. 3.11.

[4] J. I. Gittleman and B. Rosenbloom, "Microwave properties of superconductors," Proc. IEEE **52**, 1138–1147 (1964).

[5] D. C. Mattis and J. Bardeen, "Theory of the anomalous skin effect in normal and superconducting metals," Phys. Rev. **111**, 412–417 (1958).

[6] R. B. Dingle, "The anomalous skin effect and the reflectivity of metals," Physica **19**, 311–347 (1953).

[7] The analysis equivalent to this and based on the Mattis–Bardeen theory for superconductors was first given by P. B. Miller, "Surface impedance of superconductors," Phys. Rev. **118**, 928–934 (1960).

[8] The first two integrals are standard, while the last two definite integrals are found in D. B. Haan, *Nouvelles tables d'integrales definies* (Stechert, New York, 1939), Tables 134(2) and 247(3). The correct range of validity for the last integral is $1 < p < 2$.

[9] P. B. Miller, "Surface impedance of superconductors," Phys. Rev. **118**, 928–934 (1960).

[10] J. P. Turneaure, "Microwave measurements of the surface impedance of superconductive tin and lead," Ph.D. thesis, Stanford University, 1967, available from University Microfilm, Inc. Ann Arbor, MI, 67-17, 517 [J. P. Turneaure and I. Weissman, J. Appl. Phys. **39**, 4417 (1968)].

[11] J. W. MacDougall, "An experiment on the skin effect," Am. J. Phys. **44**, 978–980 (1976).

[12] J. R. Gosselin, P. Rochon, and N. Gauthier, "Study of eddy currents in a cylindrical wire: An undergraduate laboratory experiment," Am. J. Phys. **50**, 440–443 (1982).

[13] H. D. Wiederick and N. Gauthier, "Frequency dependence of the skin depth in a metal cylinder," Am. J. Phys. **51**, 175–176 (1983).

[14] L. O. Juri, V. I. Bekeris, and R. G. Steinman, "Skin depth and complex magnetic susceptibility: An experimental alternative approach," Am. J. Phys. **54**, 836–837 (1986).

Relativistic aspects of nonrelativistic quantum mechanics

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It is argued that nonrelativistic quantum mechanics does not in all respects behave as a fully Galilean invariant theory. The difference is empirically significant, as is illustrated by the Sagnac effect. The conclusion of this article sheds some new light on a recent discussion concerning the status of de Broglie's theory of matter waves.

I. INTRODUCTION

When Louis de Broglie introduced the concept of matter waves, he made essential use of special relativity.[1] His fundamental argument was that quantum theory associates a frequency with any energy, according to the formula $E = h\nu$. If the relativistic energy $E_0 = m_0c^2$ is substituted, we get, for the frequency associated with a resting particle of rest mass m_0, $\nu_0 = m_0c^2/h$. When the same particle is described from an inertial frame that has a velocity u with respect to the particle's rest frame, its energy becomes $E = m_0c^2/\sqrt{1 - u^2/c^2}$. In a Lorentz invariant theory, we should have the same relation between frequency and energy in every inertial frame, so that the frequency has to transform as $\nu' = \nu_0/\sqrt{1 - u^2/c^2}$. But this is the transformation formula for the frequency of a wave that is extended in space (as opposed to the frequency of a periodic phenomenon located *in* the particle; the latter frequency would transform as $\nu' = \nu_0\sqrt{1 - u^2/c^2}$).

In this way de Broglie was led to associate an "accompanying phase wave" with every particle. He assumed that the wave has everywhere the same phase if seen from the particle's rest frame; from the transformation behavior of the wave under Lorentz transformations, it then follows that for a particle with velocity v, the phase wave has frequency $m_0c^2/h\sqrt{1 - v^2/c^2}$, wavelength $h\sqrt{1 - v^2/c^2}/m_0v$, and phase velocity c^2/v.

When Schrödinger shortly afterward developed his wave mechanics, he built on the foundations laid by de Broglie.[2] But Schrödinger's treatment was nonrelativistic. He considered waves with frequency $mv^2/2h$ and with wavelength h/mv (in the case of free particles), for which his famous wave equation holds.

Schrödinger's nonrelativistic quantum mechanics had an enormous success; as a small part of its many accomplishments, it reproduced the exploits of de Broglie's theory. In its applications no appeal to the theory of relativity is necessary. It is somewhat surprising, and important to note, that although the results of de Broglie's relativistic approach and the later nonrelativistic treatment are almost the same, the reasoning involved is sometimes quite different. Most notably, in de Broglie's theory the frequency associated with a particle is always very high, due to the large value of the rest energy m_0c^2. In de Broglie's view this constituted an essential part of his theory. But in the Schrödinger theory there is no such "rest frequency."

The differences in the relativistic and nonrelativistic approaches to wave mechanics have given rise to various reactions in the literature. It has been alleged that the nonrelativistic version of de Broglie's phase waves is inconsistent, and that this has grave consequences for the scientific status of quantum mechanics.[3] On the other hand, it has recently also been contended that a confusion on de Broglie's part is at the heart of the idea of relativistic phase waves, and that only the nonrelativistic version can claim to be a successful physical theory.[4] According to this criticism, the high relativistic frequency of de Broglie's original theory is unphysical and the agreement with the later Schrödinger theory more or less coincidental. Finally, the common view in textbooks on quantum mechanics seems to be that nonrelativistic quantum mechanics is a perfectly alright Galilean invariant theory that is able to stand on its own feet, quite independently of relativistic considerations.[5]

The purpose of this article is to show that nonrelativistic quantum mechanics in a subtle way does not behave as a fully Galilean invariant theory. The phase of the wavefunction transforms in a way that is not compatible with the behavior of classical waves under Galilei transformations. This transformation behavior of the phase is empirically significant: On account of it, the Schrödinger theory is able to make predictions that have a typically relativistic character and cannot be expected from a Galilean invariant theory. Although nonrelativistic quantum mechanics is certainly an internally consistent theory in its own right, it therefore does not completely fit in with the framework of classical, prerelativistic theories. The theory can be regarded as a nonrelativistic approximation to a relativistic theory. Its transformation group, the 11 parameter quantum mechanical Galilei group[6]—which is the nonrelativistic limit ($c \to \infty$) of a representation of the Poincaré group (see Ref. 8, pp. 341–342)—is not a full symmetry group of the theory in exactly the same way the Galilei group is a symmetry group for classical theories.

II. THE TRANSFORMATION OF THE PHASE

Consider the nonrelativistic version of the theory of matter waves, according to which a particle with kinetic energy $E = \frac{1}{2}mv^2$ and with momentum $p = mv$ is associated with a wave with frequency $\nu = E/h$ and wavelength $\lambda = h/p$. Suppose that the wave propagates in the direction of the positive x axis. The phase at a point with coordinate x at time t is then given by the expression $\phi = (px - Et)/\hbar = m(vx - \frac{1}{2}v^2t)/\hbar$. If the same situation is described from a Galilean frame (x', t') that moves with a velocity $-u$ seen from the frame (x,t), we have $x' = x + ut$, $t' = t$, and $v' = v + u$. In terms of the variables in the new frame, the phase ϕ can be expressed as

$$\phi = (m/\hbar)(v'x' - \tfrac{1}{2}v'^2t' - ux' + \tfrac{1}{2}u^2t').$$

The fundamental formulas of the matter-wave theory could, however, also be applied directly in the moving frame, with the result

$$\phi' = (m/\hbar)(v'x' - \tfrac{1}{2}v'^2t').$$

Comparison of the formulas yields the transformation law for the phase:

$$\phi' = \phi + (m/\hbar)(ux' - \tfrac{1}{2}u^2t').$$

The phase in the frame (x',t') can therefore be found from the one calculated in the frame (x,t) by adding the "correction term" $m(ux' - \frac{1}{2}u^2t')/\hbar$. The phase is apparently not an invariant in the nonrelativistic theory.

In the Schrödinger formalism the same transformation law holds for the Fourier components of the wavefunction. Moreover, as the phase factor is the same for all Fourier components, the transformation law is valid for an arbitrary wavefunction—also if the particle is not free. That means that if $\Psi(x,t)$ is a solution of the Schrödinger equation in the inertial frame (x,t), the corresponding solution of the Schrödinger equation written down in the coordinates of the inertial frame (x',t') is given by

$$\Psi'(x',t') = \Psi(x' - ut,t)\exp[(im/\hbar)(ux' - \tfrac{1}{2}u^2t')].$$

For the Galilei transformations considered here, we have, of course, $t = t'$.

III. GALILEAN INVARIANCE

The transformations of the wavefunction that we have just derived belong to a (projective) representation of the group of Galilei transformations. Moreover, the Schrödinger equation is clearly invariant under the transformations. The derivation of the transformations had in fact as its main premise that solutions of the Schrödinger equation with momentum p are carried over into solutions of the same equation (but in the new coordinates) with momentum $p + mu$; from this, the general invariance of the Schrödinger equation already follows. The fact that these two conditions are fulfilled (representation of the Galilei group and form invariance of the evolution equation) is usually taken as sufficient evidence that the Schrödinger theory is Galilean invariant. The idea behind this, apparently, is that the wavefunctions are only affected by the transformations through multiplication by a phase factor, so that detection probabilities phenomena remain the same regardless which frame is used to make predictions.

But this reasoning is not cogent. A phase factor is only without physical significance in quantum mechanics if the *total* state vector in Hilbert space is multiplied by such a factor. In the case we are considering, the various $\Psi(x,t)$—which can be regarded as *components* of the total state vector on a basis of position eigenvectors—are multiplied by different phase factors. That suggests that the factors may have empirical significance; they affect the phase relations between the components of the total wavefunction. These phase relations may lead to empirical consequences in the case of interference.

In the following we shall see that there are indeed empirically verifiable interference effects related to the phase factors in the transformations of Sec. II. The existence of these effects implies that the Schrödinger theory is not a Galilean invariant theory in exactly the same way as classical theories are. The common conception that the Schrödinger theory *is* in that way Galilean invariant is based on an underestimation of the physical significance of the quantum mechanical phase.

IV. THE SAGNAC EFFECT

Consider the situation of Fig. 1. On a disk a device has been mounted that is able to simultaneously emit two signals of the same sort and with the same velocity in opposite directions. The signals are made to run along the same circular path (with center in the middle of the disk). Upon their return at the point of emission, the two signals are detected and compared. Now the disk, with the complete

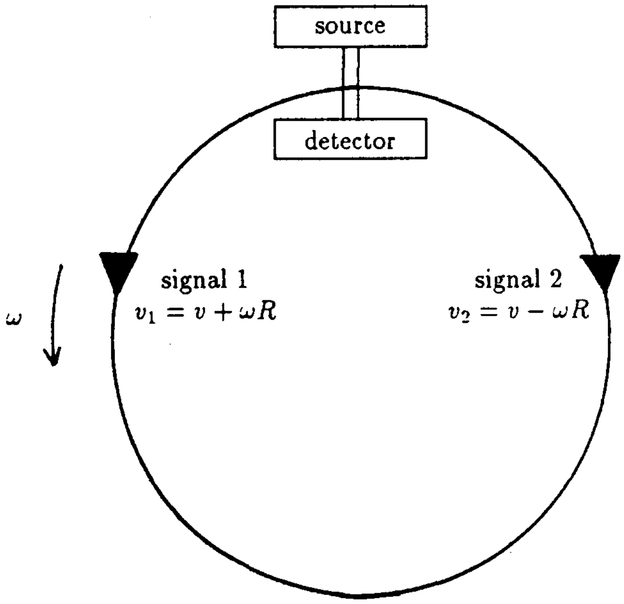

Fig. 1. The Sagnac experiment. Two signals run in opposite directions along a circular path. The whole setup can be set into rotation with respect to the laboratory frame. Seen from the rotating system, the two signals have equal speeds; measured in the laboratory system, the speeds are $v \pm \omega R$.

experimental setup attached to it, is set into rotation around its center. The question is whether the overall rotation will have an effect on what is measured by the detector.

Classical and relativistic physics predict fundamentally different things for this case. Let us first see what happens from the point of view of a Galilean invariant theory. The rotating disk is, of course, not an inertial frame, but during very short times we may regard the motion of a small portion of the circumference of the circle as uniform. The effects of centrifugal and Coriolis forces have, of course, to be compensated for in order to keep the signals on their circular trajectories. Because the acceleration $\mathbf{a}$ is then always perpendicular to the velocity, the applicability of physical laws in their inertial form is especially clear in this particular case: A possible deviation from the inertial form would result through operation of $\mathbf{a}\cdot\partial/\partial\mathbf{v}$ on the inertial functional form.

From Galilean invariance it is obvious that a comoving observer on the disk sees the departure of two oppositely directed signals with equal speeds. After a full revolution the signals meet at the position of the source; the time needed is $2\pi R/v$, with R the radius of the circle and v the speed of the signals. This equals the travel time when the disk rests in an inertial system. Everything happens at the detector as if there were no rotation.

Of course, the experiment can also be described from the inertial system with respect to which the disk has been made to rotate. Then, the two signals have speeds $v + \omega R$ and $v - \omega R$, respectively (ω is the angular velocity of the disk). But this difference in speed is exactly compensated by the difference in the distances that the signals have to travel. The faster signal has to make more than one revolution, as seen from the laboratory frame, to catch up with the detector, whereas the slower signal needs less than one revolution because the detector is moving toward it. As a result, the signals arrive simultaneously.

If the signals are waves, there is still the question of whether their phase relation is affected by the rotation. For a Galilean invariant-wave theory (e.g., of sound waves in a medium that is also brought into rotation), the answer is again negative. For a comoving observer everything remains the same as in an inertial system. Reasoning from the (inertial) laboratory frame, we reach the same conclusion in a slightly more complicated way. The two signals then have different frequencies as a consequence of the Doppler effect: $\Delta\nu = \pm\omega R/\lambda$. The wavelength λ does not change in the transition from one system to another. The time needed to reach the detector is $T = 2\pi R/v$, with v the group velocity of the waves. In this time the faster signal has traveled the distance $l_1 = 2\pi R(v + \omega R)/v$ and the slower signal $l_2 = 2\pi R(v - \omega R)/v$, so that there is a difference $\Delta l = \pm 2\pi\omega R^2/v$ compared with the static situation. The total phase shift incurred by the individual signals as a consequence of the rotation is therefore

$$2\pi\left(\frac{\Delta l}{\lambda} - \Delta\nu\, T\right) = \pm 2\pi\left(\frac{2\pi\omega R^2}{v}\frac{1}{\lambda} - \frac{\omega R}{\lambda}\frac{2\pi R}{v}\right) = 0.$$

More generally, any theory that is fully Galilean invariant, in the sense that for the comoving observer all physically relevant parameters are the same as for a corresponding observer in rest, will not predict an observable effect of the rotation.

Now, consider the relativistic version of the above arguments. Again, we treat the circular motion as uniform during very short time intervals; we have to use Lorentz transformations to relate the description in the momentarily comoving Lorentz frames to the one valid in the laboratory frame. For a Lorentz invariant theory, the equations a comoving observer can use to describe the propagation of the signals are the same as the equations that apply in a rest frame. So the observer attached to the rotating disk again sees the departure of two signals with oppositely directed velocities; at all times the velocities of the two signals, in their respective momentary inertial frames, are v and $-v$. This is the same situation as in the classical theory, and perhaps one would be inclined to predict on this basis that there will be no observable effect of the overall rotation. But in fact there is such an effect. This typically relativistic phenomenon is a consequence of the fact that time is not absolute according to the theory of relativity. If x and t denote the coordinates of a momentary inertial frame moving along with a segment of the moving disk (x measured along the circular path of the signals), we have the following relation between t and the time t' in the laboratory frame:

$$t' = (t \pm \omega R x/c^2)/\sqrt{1 - \omega^2 R^2/c^2}.$$

This is just the Lorentz transformation; the plus or minus sign applies dependent on whether the direction of increasing x is the same as or opposite to the direction of motion of the circular segment. For the signal that moves in the same direction as the rotating disk, we thus find, for the time $\Delta t'_1$ needed for one complete revolution (from source to detector),

$$\sqrt{1 - \frac{\omega^2 R^2}{c^2}}\,\Delta t'_1 = \int dt + \frac{\omega R\, 2\pi R}{c^2\sqrt{1 - \omega^2 R^2/c^2}},$$

and for the other signal,

$$\sqrt{1 - \frac{\omega^2 R^2}{c^2}}\,\Delta t'_2 = \int dt - \frac{\omega R\, 2\pi R}{c^2\sqrt{1 - \omega^2 R^2/c^2}}.$$

The integral $\int dt$ represents the total time as judged from the successive momentary inertial frames; it equals $2\pi R/v\sqrt{1-\omega^2R^2/c^2}$. It must be kept in mind that Euclidean geometry is not valid on the rotating disk; the circumference of the circle as measured with measuring rods resting on the disk is $2\pi R/\sqrt{1-\omega^2R^2/c^2}$ on account of the Lorentz contraction. This is the origin of the square root at the right-hand side of the above formulas.

Apparently, the two signals do not arrive simultaneously at the detector. In the coordinate time of the laboratory frame, the difference in arrival times is

$$\Delta t_1' - \Delta t_2' = 4\pi R^2\omega/(c^2 - \omega^2R^2).$$

The difference is $\Delta t = 4\pi R^2\omega/c^2\sqrt{1-\omega^2R^2/c^2}$, if measured by a comoving clock at the position of the detector. The existence of the difference is due to the fact that the momentary inertial frames do not mesh together to form one global frame with standard simultaneity. Although the proper time interval from source to detector is the same for the two signals [it equals $2\pi Rv^{-1}\sqrt{(1-v^2/c^2)/(1-\omega^2R^2/c^2)}$], this does not mean that the time interval is the same in terms of coordinate time—and interesting variation on the notorious twin paradox.

The appearance of a difference in arrival times—and an associated phase difference—when the total system is set into rotation is known as the Sagnac effect. In the above derivation of the effect, it is transparent that the magnitude of the time difference does not depend on the speed v of the signals, but only on the angular velocity of the rotating system. This is one of the reasons that the effect provides an expedient tool to detect rotation with respect to an inertial frame. If an interferometer has been mounted on the disk, a shift in the fringes at the detector is evidence for rotational motion of the system (the principle of the laser gyroscope).

The Sagnac effect was originally discussed for the case of light[7]; in this case the time difference Δt is connected with a phase difference

$$\Delta\phi = 2\pi\nu\,\Delta t = 8\pi^2R^2\nu\omega/c^2\sqrt{1-\omega^2R^2/c^2}.$$

The effect is, however, as shown in the preceding paragraphs, quite general. A rotational motion of the experimental setup always produces a difference in arrival times, independently of the type or velocity of the signal (for wave phenomena the time difference is, of course, associated with a phase difference). The origin of the effect is purely relativistic. The crucial ingredient in the derivation are the relations between times as measured in inertial systems that are in relative motion.[8]

V. THE QUANTUM SAGNAC EFFECT

Let us now use the transformation formulas for the quantum mechanical phase (Sec. II) to see what nonrelativistic quantum mechanics predicts for the results of an experiment of the Sagnac type. According to the Schrödinger theory, two particles with opposite velocities can be represented by two wave packets with opposite group velocities. On a disk that is in rest with respect to the laboratory frame, the two packets, of course, arrive simultaneously at the detector. The question now is whether anything observable will change when the whole experimental arrangement is set into uniform rotation.

Clearly, the rotation has no effect on the time needed by the wave packets to reach the detector. The velocities transform according to a Galilei transformation, and this implies that for an observer in rest with respect to the rotating disk the travel times are the same as when the disk is resting in the laboratory frame. But for the phases there *is* a difference. The observer on the disk should now *not* perform the calculations by directly using the expression for the de Broglie phase, $\phi = (px - Et)/\hbar$, everywhere in the frame, but should take the "correction term" $m(ux' - \frac{1}{2}u^2t)/\hbar$ into account in order to relate the results to phase calculations in the laboratory frame. The reason is that the corresponding phase factor assumes different values for the two wave packets, which gives rise to a phase shift at the position of the detector. Indeed, as the relative directions of increasing x and u are different for the two packets, there is a total difference in phase at the detector of $2m\omega R\, 2\pi R/\hbar = 4\pi R^2 m\omega/\hbar$. This equals, apart from a relativistic correction factor $1/\sqrt{1-\omega^2R^2/c^2}$, the phase shift to be expected on the relativistic theory. Relativistically, there is a difference in arrival times of the packets $\Delta T \approx 4\pi R^2\omega/c^2$, and in view of the relativistic frequency of the matter waves $\nu = mc^2/h$ this leads to a phase shift $\Delta\phi = 2\pi\,\Delta T\,\nu = 4\pi R^2 m\omega/\hbar$. The same expression is found if the frequency–mass relation $\nu = mc^2/h$ is inserted in the formula for the phase shift in the case of light (Sec. IV). It is worth noting that in the relativistic derivation, essential use is made of the very high frequency mc^2/h of the matter waves. Only because of the high frequency, proportional to c^2, is there a phase difference of zeroth order in c^{-1} (ΔT is of order c^{-2}). The nonrelativistic theory reproduces the phase difference, but without ascribing the relativistic frequency to the waves and without predicting a difference in arrival times.

The calculation of the phase shift can also be carried out directly in the laboratory frame. We can then use the expression for the phase $\phi = (px - Et)/\hbar$; one wave packet is associated with a momentum $m(v + \omega R)$ and energy $\frac{1}{2}m(v+\omega R)^2$, and the other with momentum $m(v - \omega R)$ and energy $\frac{1}{2}m(v-\omega R)^2$. In other words, there is a Doppler effect not only on the frequency, but also on the *wavelength*—already an indication that we are not dealing with a Galilean invariant theory. The time needed by the wave packets to reach the detector is the same: $T = 2\pi R/v$; the traversed distance is $(v \pm \omega R)T$. The first packet then acquires a phase

$$\phi_1 = \frac{1}{\hbar}\left(m(v+\omega R)^2T - \frac{1}{2}m(v+\omega R)^2T\right)$$
$$= m(v+\omega R)^2T/2\hbar,$$

and the second one a phase $\phi_2 = m(v-\omega R)^2T/2\hbar$. The phase difference consequently is

$$\phi_1 - \phi_2 = 4\pi R^2 m\omega/\hbar.$$

Note that direct application of the Schrödinger theory everywhere in the comoving frame, with $E = h\nu$ and $p = h/\lambda$, would have led to the wrong result that there is no Sagnac phase shift.

The phase shift gives rise to an observable effect. If Ψ and Φ designate the wave packets in the situation without rotation,we find for the total wavefunction at the detector, if the disk is rotating, $k(\Psi e^{i\phi_1} + \Phi e^{i\phi_2})$, with k a normalization constant. The probability to detect a particle is consequently proportional to $|\Psi|^2 + |\Phi|^2 + 2\,\mathrm{Re}\,\Psi\Phi^* e^{i(\phi_1-\phi_2)}$. The Sagnac phase shift is therefore reflected in the counting rate of particles in the detector. The effect has been verified experimentally in neutron interferometry, where the rotation of the Earth has proved to be detectable in this way.[9]

In summary, nonrelativistic quantum mechanics predicts a Sagnac effect. It was argued in Sec. IV that a Galilean invariant theory, in which all inertial frames are equivalent and can be used alternately, cannot entail such an effect. The Schrödinger theory (and the nonrelativistic version of the de Broglie theory of matter waves) can therefore not be fully Galilean invariant in this sense. The preceding calculations also make it clear where in the formalism the nonGalilean characteristics have to be located. The phase of the wavefunction transforms in a nonclassical way, and in some situations this can be the origin of observable effects. In such circumstances not all frames that are related by Galilei transformations to the inertial frame of the laboratory are physically equivalent . In the case of the rotating disk, the expression for the phase $\phi = (px - Et)/\hbar$ can be applied in the laboratory frame, but it can not be correct in *all* momentary Galilean frames moving along with the periphery of the disk. This does not mean that there is a *preferred* frame of reference. The de Broglie phase relations can be applied in any one chosen local inertial system, but not in all these systems at once. The lack of complete Galilean invariance can be seen as resulting from the fact that the phase of the nonrelativistic de Broglie or Schrödinger waves is an approximation (to zeroth order in c^{-1}) of the relativistic phase, as will be shown in Sec. VI.

The situation closely parallels the one encountered in the theory of electrodynamics. In the presence of an externally given four-vector field A^{μ}, the wavefunction acquires a phase factor,which can be transformed away locally by a gauge transformation—just as the phase shift can be made to vanish in our case by means of the choice of the local inertial system in which the de Broglie relations are to hold. However, the electromagnetic phase shift usually cannot be transformed away *everywhere* along a closed loop; the phase factor is nonintegrable, as demonstrated in the Aharonov–Bohm effect. The requirement that there should be an underlying theory that is invariant under local gauge transformations then leads to the full Maxwell theory, in which the field A^{μ} is dynamically coupled to the electrical charge. In the same way the existence of the Sagnac phase shift points into the direction of an underlying *gravitational* theory, containing a coupling to the mass rather than to the electrical charge, which dynamically determines the inertial systems. Following up this analogy here would take us too far afield, however.[10]

VI. CONCLUSION

In the relativistic theory, the Sagnac effect for the phase of matter waves is in a sense "doubly relativistic": It results from the combination of two typically relativistic features. First, there is the time difference $\Delta T \approx 4\pi R^2\omega/c^2$; second, there is the frequency associated with the energy mc^2: $\nu = mc^2/h$. Together this gives the phase shift $4\pi R^2 m\omega/\hbar$. How is it possible that the nonrelativistic theory, which operates with the much lower frequency $\nu = \frac{1}{2}mv^2/h$, yields the same phase shift? Moreover, how is it possible that the nonrelativistic theory predicts a Sagnac effect at all? The answer is that the nonrelativistic de Broglie and Schrödinger theories are not in all respects full-blown Galilean invariant theories. They can be seen as nonrelativistic approximation schemes to a relativistic, Lorentz invariant theory. The details of the approximation clearly come out if one looks at how the Schrödinger equation follows as a limiting case from, e.g., the Dirac equation.[11] The first step in the transition to the classical limit consists in the multiplication of the Dirac wavefunction with the phase factor $\exp(im_0c^2t/\hbar)$, to take account of the relativistic rest energy. It is then shown that the resulting wavefunction (or rather the first two of its four components) obeys, in first approximation, a Schrödinger-type of equation. This means that the phase in the Schrödinger theory equals, to zeroth order in c^{-1}, the relativistic phase minus $m_0c^2t/\hbar$. Phase *differences* at one moment are not affected by this subtraction procedure. That explains how the nonrelativistic theory, although working with a completely different frequency, can predict the same phase relations as the relativistic theory. However, the approximation procedure is not Galilean, but Lorentz invariant. If we go from an inertial frame (x,t) to an inertial frame (x', t'), with $t' = (t + ux/c^2)/\sqrt{1-u^2/c^2}$, $x' = (x+ut)/\sqrt{1-u^2/c^2}$, and then define the "Galilean coordinates" $t'_G = t$ and $x'_G = x + ut$, we find for the phase in these new Galilean coordinates, after subtraction of $m_0c^2t/\hbar$, and neglecting terms of order (v/c) and higher, exactly the expression of Sec. II, $\phi' = \phi + m(ux'_G - \frac{1}{2}u^2t'_G)/\hbar$, with $\phi' = (p'x'_G - E't'_G)/\hbar$ and the nonrelativistic expressions for p' and E'. This is, of course, not surprising. But this way of deriving the expression shows how the "strange" correction term in the nonrelativistic theory results from approximating the Lorentz invariant phase of the relativistic theory in terms of the "wrong," i.e., Galilean, coordinates. The contributions to the correction term, although of zeroth order in c^{-1}, have a typically relativistic character; one contribution, e.g., results from the multiplication of the term ux/c^2 from the relativistic time transformation with the relativistic frequency mc^2/h.

Our conclusion is that nonrelativistic quantum mechanics contains relativistic elements: It is able to make predictions about observable effects that are typically relativistic. The essential background of this is the fact that the fundamental formulas of de Broglie's nonrelativistic wave mechanics cannot hold in *all* Galilei frames of reference *at once*. Although the quantum mechanical Galilei group leaves the Schrödinger equation invariant in the transition from one inertial frame to another, it is therefore not an invariance group in exactly the same way as the Galilei group is for classical theories.

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[b] Department of Atomic Physics.

[1] Louis de Broglie, "Ondes et quanta," C. R. Acad. Sci. **177**, 507–510 (1923). De Broglie gave an extensive and systematic discussion of his ideas in his thesis "Recherches sur la Théorie des Quanta," published in its entirety in Ann. Phys. (Paris) **3**, 22–128 (1925).

[2] E. Schrödinger, "Quantisierung als Eigenwertproblem (Erste Mitteilung)," Ann. Phys. (Leipzig) **79**, 361–376 (1926); "Quantisierung als Eigenwertproblem (Zweite Mitteilung)," **79**, 489–527 (1926). See also E. Schrödinger, "Zur Einsteinschen Gastheorie," Phys. Z. **27**, 95–101 (1926) and Linda Wessels, "Schrödinger's route to wave mechanics," Stud. Hist. Philos. Sci. **10**, 311–340 (1977).

[3] Alfred Landé, "Quantum fact and fiction III," Am. J. Phys. **37**, 541–548 (1969). See also the reaction by P. C. Peters, "Consistency of the de

Broglie relations with special relativity," Am. J. Phys. 38, 931–932 (1970).
[4] Edward MacKinnon, "De Broglie's thesis: A critical retrospective," Am. J. Phys. 44, 1047–1055 (1976). MacKinnon's conclusions are criticized by Manfred Stöckler, *Philosophische Probleme der relativistischen Quantenmechanik* (Duncker & Humblot, Berlin, 1984).
[5] See, for instance, L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Non-Relativistic Theory* (Pergamon, London, 1959) or A. Messiah, *Quantum Mechanics* (North-Holland, Amsterdam, 1962), Vol. II.
[6] Jean-Marc Lévy-Leblond, "Galilei group and Galilean invariance," in *Group Theory and Its Applications*, edited by Ernest M. Loebl (Academic, New York, 1971), Vol. II, pp. 221–299.
[7] G. Sagnac, "Les systèmes optiques en mouvement et la translation de la Terre," C. R. Acad. Sci. 152, 310–313 (1911); "L'éther lumineux démontré par l'effet du vent relatif d'éther dans un interféromètre en rotation uniforme," C. R. Acad. Sci. 157, 708–710 (1913); "Sur la preuve de la réalitè de l'éther lumineux par l'expérience de l'interférographe tournant," C. R. Acad. Sci. 157, 1410–1413 (1913).
[8] A profound theoretical treatment of the Sagnac effect, also for matter waves, can be found in J. Anandan, "Sagnac effect in relativistic and nonrelativistic physics," Phys. Rev. D 24, 338–346 (1981).
[9] S. A. Werner, J. -L. Staudenmann, and R. Colella, "Effect of Earth's rotation on the quantum mechanical phase of the neutron," Phys. Rev. Lett. 42, 1103–1106 (1979).
[10] The analogy between the Coriolis force $2m\mathbf{v}\times\omega$ and the Lorentz force $e\mathbf{v}\times\mathbf{B}$ can be used to read off the form of the Sagnac phase shift from the form of the phase shift that occurs if the interfering paths enclose a magnetic flux. See further on this analogy: J. Anandan, "Gravitational and rotational effects in quantum interference," Phys. Rev. D 15, 1448–1457(1977); J. J. Sakurai, "Comments on quantum-mechanical interference due to the Earth's rotation," Phys. Rev. D 21, 2993–2994 (1980).
[11] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

Electromechanical analog for Landau's theory of second-order symmetry-breaking transitions

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A simple electromechanical analog for Landau's classical theory of second-order symmetry-breaking transitions is presented. The apparatus consists of a torsion pendulum, where a ceramic magnet suspended by two torsion springs is subjected to a uniform magnetic field. The pendulum angle is the analog of the order parameter in a phase transition, while the current passing through the Helmholtz coil, which produces the magnetic field, plays the role of temperature. With this setup, the critical exponents β, δ, γ, and γ' can be determined with good accuracy.

I. INTRODUCTION

One of the fundamental theories concerning the nature of changes in thermodynamic quantities in a phase transition between phases of different symmetries was introduced by Landau.[1,2] He was the first to emphasize the fundamental importance of symmetry in phase transitions.[3] The ferroelectricity of barium titanate, for instance, is a well-known example of a symmetry-breaking transition: As the temperature is reduced and passes through the critical point, there is a transition to a lower crystal symmetry (cubic to tetragonal), which is accompanied by the sudden appearance of a macroscopic lattice polarization.[4] Apart from this canonical example, there are many other critical phenomena such as liquid–solid transitions, ferromagnetism, superconductivity, and superfluidity that can, as in the case of the ferroelectricity in barium titanate, be described in terms of symmetry-breaking transitions.

In critical phenomena involving broken symmetry, the less symmetric phase is usually characterized by an order parameter η with a nonzero value, which may represent the dielectric polarization in ferroelectrics, the magnetization in ferromagnets, etc. On the other hand, the value of η is zero in the symmetric phase. During the phase transition, whenever the value of η goes from zero to a finite value in a continuous way, we have a second-order transition; otherwise, the transition is a first-order one, like the liquid–solid transition.

The mathematical treatment given by Landau to a second-order transition is a simple and very elegant speculation concerning a possible universal behavior of a thermodynamic potential near the critical point. However, this approach fails in many cases, and the reason for this failure is related to Landau's assumption about the existence of a particular series expansion for the thermodynamic potential. The so-called classical exponents, which are a direct consequence of these assumptions, are not in agreement with experimental values in many substances. The disagreement is particularly noticeable in systems that appear to have an infinite specific heat at the critical point. However, Landau's theory still provides a reasonable description of systems like ferroelectrics and superconductors in which the specific heat does not diverge at the critical point.

In this work we present an electromechanical system showing an inversion symmetry-breaking transition, whose potential energy has the same critical behavior predicted by Landau's theory. In contrast to previous mechan-

