Simple thermodynamics of jet engines

Pedro Patrício[a)] and José M. Tavares
Instituto Superior de Engenharia de Lisboa, Rua Conselheiro Emídio Navarro 1,
P-1949-014 Lisboa, Portugal and Centro de Física Teórica e Computacional, Universidade de Lisboa,
Avenida Professor Gama Pinto 2, P-1649-003 Lisboa Codex, Portugal

We use the first and second laws of thermodynamics to analyze the behavior of an ideal jet engine. Simple analytical expressions for the thermal efficiency, the overall efficiency, and the reduced thrust are derived. We show that the thermal efficiency depends only on the compression ratio r and on the velocity of the aircraft. The other two performance measures depend also on the ratio of the temperature at the turbine to the inlet temperature in the engine, T_3/T_i. An analysis of these expressions shows that it is not possible to choose an optimal set of values of r and T_3/T_i that maximize both the overall efficiency and thrust. We study how irreversibilities in the compressor and the turbine decrease the overall efficiency of jet engines and show that this effect is more pronounced for smaller T_3/T_i. © *2010 American Association of Physics Teachers.*
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I. INTRODUCTION

The first and second laws are usually illustrated for closed systems. Thermodynamics textbooks[1–3] discuss the Carnot cycle and other closed cycles (for example, the Rankine, Otto, Diesel, and Brayton cycles) and calculate their ideal performance based on the assumption of reversible transformations. Many of the machines used to transfer thermal energy (even those that are examples of closed cycles) contain fluids that exchange energy with their surroundings. Jet engines, which can be viewed as an assemblage of several of these devices, provide an important and interesting application of thermodynamic laws to open stationary systems. However, most introductory thermodynamics textbooks do not discuss such engines from a general perspective.

The purpose of this paper is to analyze a generic jet engine and show the simplicity, power, and the beauty of the laws of thermodynamics. We will find that it is possible to understand the nature of a ramjet, the role of the turbine and the compressor and why increasing the compression ratio and developing turbines able to withstand high temperatures were important in the development of jet engines for commercial aircraft. We will also understand how this development was affected by the constraints imposed by the second law.

This paper is organized as follows. First, we introduce the basic concepts of thrust and of overall propulsive and thermal efficiencies, which will help us to measure the performances of jet engines. Next, we will describe the simplest possible jet engine and analyze its thermal efficiency. We will then consider an engine with a turbine and a compressor and study their affect on the thermal efficiency. We then analyze the overall efficiency and the thrust, which depend also on the maximum temperature attained. We discuss the consequences of this dependence for finding the best design for a jet engine. We then investigate the effects of irreversibilities in the compressor and turbine on the overall efficiency. Finally, we will discuss ways to introduce the results of this paper to students.

II. PERFORMANCES OF JET ENGINES

The motion of an aircraft is due to air propulsion. As it flies, the aircraft engine accelerates the air and, according to Newton's law, produces a thrust F, which is equal to the difference between the exit and inlet flow velocities, respectively, v_e and v_i, times the mass airflow rate $\dot{m}$ (the mass of air that is accelerated per unit time),[4]

$$F = \dot{m}(v_e - v_i). \tag{1}$$

There are two main types of engines to accelerate the air. The piston engine creates mechanical work, which is transmitted to a fan (propeller), which in turn produces the required thrust. The jet engine provides thrust by burning the air with fuel[5] in a combustion chamber and exhausting the high-temperature mixture through a nozzle that accelerates the air. Airplanes commonly combine both types of engine to benefit from their respective advantages. In this paper we will study jet engines only.

To characterize an engine's performance, we define the dimensionless or *reduced thrust* by[6]

$$\bar{F} = \frac{F}{\dot{m}v_s}, \tag{2}$$

where v_s is the local speed of sound. Usually, in aircraft the mass airflow rate $\dot{m}$ is associated with the physical size of an engine, including diameter and weight. Thus the reduced thrust is proportional to the thrust-to-weight ratio.

The overall efficiency η of an aircraft engine is defined as the ratio of the (mechanical) propulsive power to the (thermal) power obtained when fuel is burned.[6–8] If we assume that all the energy released in fuel combustion is absorbed by the air,[7] we have

$$\eta = \frac{Fv_i}{\dot{m}q}, \tag{3}$$

where q is the thermal energy per unit mass absorbed by the air. It is useful to express the overall efficiency as the product[8]

$$\eta = \eta_{\text{th}}\eta_p, \tag{4}$$

where η_{th} is the thermal efficiency, which is defined as the ratio of the rate of production of kinetic energy to the thermal power obtained when the fuel is burned,
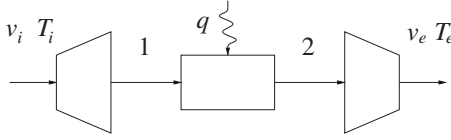
Fig. 1. A simple model of a jet engine. The air enters the engine through a diffuser, goes to the combustion chamber, and is exhausted through a nozzle.

$$\eta_{\text{th}} = \frac{\Delta e_c}{q}. \tag{5}$$

The propulsive efficiency η_p is defined as the ratio of propulsive power to the rate of production of kinetic energy,

$$\eta_p = \frac{F v_i}{\dot{m} \Delta e_c}. \tag{6}$$

In Eqs. (5) and (6) Δe_c represents the change in the kinetic energy of the air per unit mass,

$$\Delta e_c = \tfrac{1}{2}(v_e^2 - v_i^2). \tag{7}$$

III. THE SIMPLEST MODEL OF A JET ENGINE

Figure 1 illustrates the simplest model of a jet engine. The outside air is at temperature T_i, and its speed relative to the aircraft, the inlet velocity, is v_i. The air enters the engine through a diffuser, which lowers its speed and increases its pressure. The air then goes into the combustion chamber, and each unit mass of air absorbs energy q, increasing its internal energy. Finally, the air is exhausted through a nozzle, which accelerates the air until it attains the exit velocity v_e at temperature T_e.

To analyze this engine, we will apply the same equation to the diffuser, the combustion chamber, and the nozzle. The relation

$$q - w = \Delta h + \Delta e_c \tag{8}$$

expresses conservation of energy for open stationary systems with a constant mass flow rate.[2] Here, w is the work per unit mass performed by the air inside the open system of interest ($w=0$ for the diffuser, the combustion chamber, and the nozzle in which there are no movable parts), Δh is the difference between the exit and inlet specific enthalpies, and Δe_c is the difference between the exit and inlet kinetic energies per unit mass. Usually, Eq. (8) also includes another term accounting for the potential energy, which we will neglect here.

If we apply Eq. (8) to the jet engine and assume that the air has a constant-pressure specific heat c_P, the kinetic energy increase is given by

$$\Delta e_c = \tfrac{1}{2}(C_e^2 - C_i^2) = q - \Delta h = q - c_P(T_e - T_i). \tag{9}$$

This result means that if the exit temperature is the same as the initial external temperature, all the thermal energy that is given to the system in the combustion chamber will be used to increase the kinetic energy of the air, leading to maximum thrust.

However, the second law of thermodynamics implies that the complete conversion of thermal energy into work (or into kinetic energy of the air) is impossible and that the most favorable situation (maximum production of work) is real-

ized if all the processes are reversible. In the subsequent analysis we will assume that all the transformations experienced by the air in the jet engine are reversible. The adiabatic and reversible transformation of the air in the nozzle leads to the relation

$$T_e = T_2 \left(\frac{P_2}{P_e} \right)^{(1-\gamma)/\gamma}, \tag{10}$$

where P_e is the atmospheric pressure outside (which then equals P_i), T_2 and P_2 are the temperature and pressure of the air upstream of the nozzle (see Fig. 1), and γ is the adiabatic coefficient of air, which we assume to be constant to simplify our calculations. A similar relation may be found for the adiabatic reversible diffuser

$$T_i = T_1 \left(\frac{P_1}{P_i} \right)^{(1-\gamma)/\gamma}, \tag{11}$$

where T_1 and P_1 are the temperature and pressure of the air downstream of the diffuser (see Fig. 1). If we assume an ideal reversible process in the combustion chamber (there are no frictional forces inside it), the air flows with no pressure drop, and $P_1 = P_2$.[9] We then combine Eqs. (10) and (11) to obtain

$$T_e = \frac{T_2}{T_1} T_i. \tag{12}$$

The temperatures T_1 and T_2 may be calculated as a function of T_i from conservation of energy, Eq. (8), applied to the diffuser and to the combustion chamber. If we neglect the relative velocities of the air inside the engine, the temperature after the diffuser is

$$T_1 = T_i + \frac{1}{2} \frac{v_i^2}{c_P} = T_i(1 + \epsilon), \tag{13}$$

where the dimensionless parameter ϵ is given by

$$\epsilon = \frac{v_i^2}{2 c_P T_i}. \tag{14}$$

This reduced kinetic energy[7] of the aircraft is related to the Mach number through

$$M = \frac{v_i}{v_s} = \frac{v_i}{\sqrt{\gamma r_g T_i}} = \sqrt{\frac{2 c_V}{r_g}} \sqrt{\epsilon} \approx 2.24 \sqrt{\epsilon}, \tag{15}$$

where $v_s = \sqrt{\gamma r_g T_i}$ is the speed of sound in the air at temperature T_i, c_V is the air specific heat at constant volume, $r_g = c_P - c_V$, and $\gamma = c_P/c_V$. We have used the numerical values $c_V = 718$ J/(kg K) and $r_g = 287$ J/(kg K) (which give $\gamma = 1.4$), applicable for air at $T_i = 300$ K. The temperature after the combustion chamber is related to T_1 by

$$T_2 = T_1 + \frac{q}{c_P}. \tag{16}$$

If we substitute these expressions for the temperatures into Eq. (9), we obtain a simple result for the thermal efficiency of a simple jet engine:

$$\eta_{\text{th}} = \frac{\Delta e_c}{q} = 1 - \frac{1}{1 + \epsilon} = \frac{\epsilon}{1 + \epsilon}. \tag{17}$$

This efficiency depends only on M and is plotted in Fig. 2 (bottom line). We can see that there is no increase in kinetic
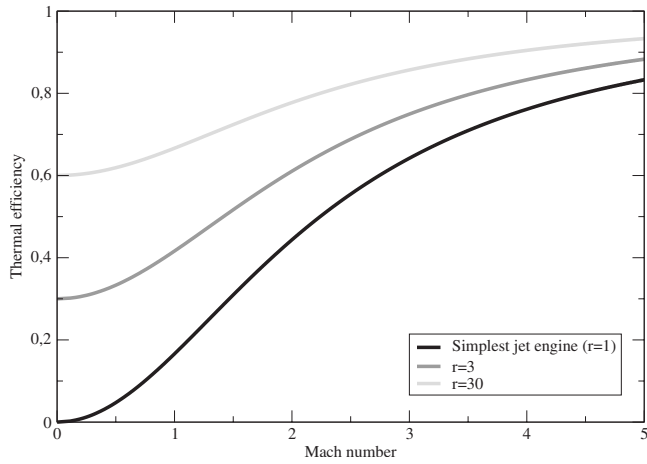
Fig. 2. The thermal efficiency η_{th} as a function of its velocity $M = v_i / v_s$ (Mach number) for the simplest jet engine with no compressor and for pressure ratios $r \approx 3$ ($a = 0.7$) and $r \approx 30$ ($a = 0.4$).

energy for an aircraft at rest ($M = 0$). Therefore, a propeller is needed to start an aircraft with this simple jet engine. For small ϵ, or velocities below the speed of sound, the thermal efficiency is approximately $\eta_{th} = \epsilon < 20\%$. Only for very high velocities, several times larger than the speed of sound, does the thermal efficiency approach unity.

The simple model of a jet engine we have described, without a compressor and turbine, corresponds to the ideal ramjet.

IV. JET ENGINE WITH AN IDEAL COMPRESSOR AND TURBINE

To achieve a much higher thermal efficiency for small velocities, we must add a compressor before the combustion chamber (that is, upstream). The work used to drive the compressor is generated by a turbine placed just after (that is, downstream) the combustion chamber (see Fig. 3). If we apply Eq. (8) to the compressor and the turbine and if energy losses are neglected, this work (associated with the rotating shafts) may be related to the temperature differences upstream and downstream from these devices,

$$w = c_P(T_4 - T_3) = - c_P(T_2 - T_1). \tag{18}$$

Because the transformations undergone by air in the compressor and the turbine are also assumed to be adiabatic and reversible, we have

$$T_4 = T_3 \left(\frac{P_4}{P_3} \right)^{(1-\gamma)/\gamma}, \quad T_1 = T_2 \left(\frac{P_2}{P_1} \right)^{(1-\gamma)/\gamma}. \tag{19}$$

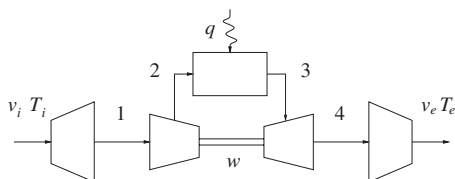The compression ratio $r = P_2 / P_1$ depends on the characteristics of the compressor.

Fig. 3. Jet engine with a compressor and a turbine.

If we use the results obtained in Sec. III, the thermal efficiency of the jet engine can now be written as

$$\eta_{th} = \frac{\Delta e_c}{q} = 1 - \frac{a}{1 + \epsilon}, \tag{20}$$

where $0 < a = r^{(1-\gamma)/\gamma} \leq 1$, because both the compression ratio r and the adiabatic coefficient γ are greater than one.

Although the engine consists of five components, the final result for its thermal efficiency, Eq. (20), is simple and depends only on a (or r) and ϵ.[10] If $r = a = 1$, there is no compression, and we obtain the thermal efficiency of the simple jet engine [see Eq. (17)]. Early jet engines used in World War II had an overall pressure ratio slightly greater than $r = 3$ ($a = 0.71$). For the Boeing 747 $r = 30$ ($a = 0.38$) and $r = 40$ ($a = 0.35$) for the Airbus A380, which for regulatory reasons do not exceed the speed of sound. It is also instructive to know that $r = 15$ ($a = 0.46$) for the Concorde.

For comparison, we present the thermal efficiencies of these jet engines in Fig. 2 for $a = 0.7$ ($r \approx 3$) and $a = 0.4$ ($r \approx 30$). For small ϵ, we may write $\eta_{th} = (1 - a) + a\epsilon$. At rest, the thermal efficiency substantially improves as r is increased (as is illustrated for $\eta_{th} = 0.3$ and $\eta_{th} = 0.6$). The initial slope of η_{th} becomes smaller as the pressure ratio increases. Nevertheless, these lines join only at infinite speeds, for which $\eta_{th} = 1$.

The results for the thermal efficiency indicate the qualitative behavior of jet engines. In particular, the thermal efficiency is a simple function of the speed of the aircraft and the compression ratio of the compressor only—no dependence on the absorbed energy was found. The thermal efficiency is the same whether we use a small or a large amount of thermal energy; the kinetic energy of the aircraft varies proportionally. According to these idealized results, if we need to travel faster, we should just use more fuel (to increase the thermal energy). However, the temperature inside the jet engine will rise with the travel speed, and we risk melting the compressor or the turbine.

V. OVERALL EFFICIENCY AND THRUST

Besides the compression ratio, the inlet temperature of the turbine T_3 (see Fig. 3), which is the highest temperature reached by air inside the engine, is the second most important parameter of the jet engine that we will take into account. This temperature is related to the absorbed energy by using the first law of thermodynamics,

$$\tilde{T}_q = \frac{q}{c_P T_i} = \frac{T_3}{T_i} - \frac{1 + \epsilon}{a}, \tag{21}$$

where we have introduced the dimensionless variable $\tilde{T}_q$. By using the definition of ϵ and Eq. (7), the propulsive efficiency can be related to η_{th} and $\tilde{T}_q$ by

$$\eta_p = \frac{2}{1 + \sqrt{1 + \frac{\eta_{th} \tilde{T}_q}{\epsilon}}}. \tag{22}$$

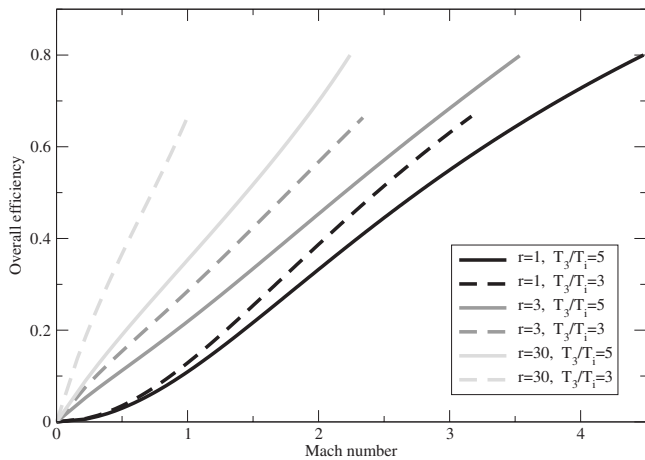The overall efficiency $\eta = \eta_{th} \eta_p$ and the reduced thrust

Fig. 4. Overall efficiency η of a jet engine as a function of its velocity M for the values of a and T_3/T_i given in the inset.

Fig. 5. Reduced thrust $\bar{F}$ as a function of M for the values of a and T_3/T_i indicated in the inset.

$$\bar{F} = \frac{F}{\dot{m}C_s} = 2.24\sqrt{\epsilon}\left[-1 + \sqrt{1 + \frac{\eta_{\text{th}}\widetilde{T}_q}{\epsilon}}\right] \qquad (23)$$

depend not only on ϵ and r (as for η_{th}) but also on T_3/T_i, through $\widetilde{T}_q$ [see Eq. (21)].

If we use Eqs. (20) and (21), we can verify that the condition $F>0$ (or $q>0$) imposes an upper bound for the thermal efficiency,

$$\eta_{\text{th}} < 1 - \frac{T_i}{T_3}, \qquad (24)$$

which is Carnot's theorem. Consequently, given a pair of the engine parameters, r and T_3/T_i, there is a velocity beyond which flight becomes impossible. This maximum velocity or, alternatively, maximum reduced kinetic energy [see Eq. (14)] is obtained by setting $\eta_{\text{th}}=1-T_i/T_3$ (or $F=0$ or $q=0$),

$$\epsilon_{\text{max}} = aT_3/T_i - 1. \qquad (25)$$

For a fixed T_3/T_i, the increase in r (decrease of a) will always lower the maximum possible velocity, and if $a < T_i/T_3$, the jet engine will not work. Therefore, we may only have engines with large compression ratios if the turbine materials can withstand high enough T_3.

The analysis of the interplay between the two parameters and their effects on the overall efficiency and reduced thrust are shown in Figs. 4 and 5, where we plot η and $\bar{F}$ [from Eqs. (20), (22), and (23)] as a function of M for several values of r and T_3/T_i.

The results in Fig. 4 confirm that increasing r (at fixed T_3 and M) increases the efficiency but only up to a certain value (the value of r for which $\epsilon=\epsilon_{\text{max}}$). It also shows that at all velocities and for fixed r, an increase in T_3/T_i decreases η. Also for fixed r and T_3/T_i, increasing M increases η. If $T_3/T_i \approx 5$ is assumed, Fig. 4 shows why the Concorde (which flew at $M \approx 2$) that was designed with $r \approx 15$ ($a \approx 0.45$) had higher efficiencies than other commercial and larger airplanes [with $M \approx 0.75$ and $r \approx 30$ ($a \approx 0.4$)].

The results presented in Fig. 5 show that the reduced thrust decreases with increasing velocity for fixed r and T_3/T_i, except for the ramjet ($r=1$), for which there is a velocity that maximizes the thrust. In this case an auxiliary
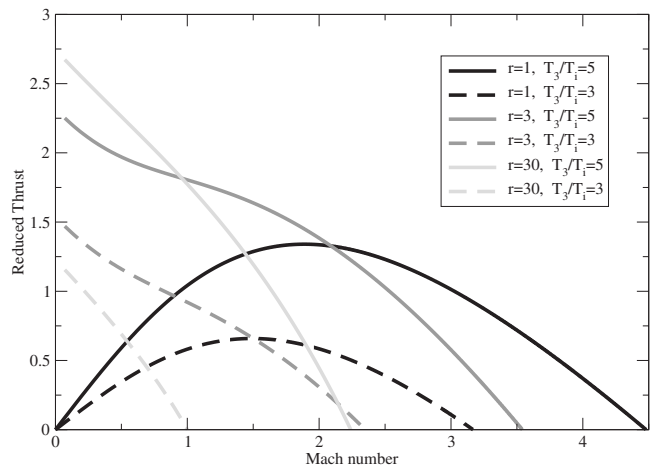
device is needed to start the motion because $\bar{F}=0$ when $M=0$. An increase in T_3/T_i increases the reduced thrust for every pair M and r. The dependence of the thrust on r for fixed T_3/T_i and M) is more complicated. If we want to use the curves in Fig. 5 to determine the value of r that maximizes the thrust, we have to consider the following cases:

(a) Low velocities ($M<1$). If $T_3/T_i=5$, the maximum thrust is obtained for $r=30$; if $T_3/T_i=3$, the maximum is for $r=3$.

(b) Intermediate velocities ($1<M<2$). If $T_3/T_i=5$, the maximum thrust is obtained for $r=3$; for $T_3/T_i=3$, the maximum thrust is obtained for $r=3$ up to $M \approx 1.5$ and $r=1$ for $M>1.5$.

(c) High velocities ($M>2$). The maximum thrust is always obtained for the ramjet $r=1$.

The results in Figs. 4 and 5 show that it is not possible to maximize the reduced thrust and efficiency at the same time due to the upper bound on the temperature T_3/T_i at the turbine. Figure 4 shows that to increase the efficiency, r should be increased to its maximum possible value. In contrast, Fig. 5 shows that the maximization of reduced thrust with a particular value of r depends on M and T_3/T_i. Therefore, the specific flight requirements of the aircraft—whether we want a fast military aircraft or an efficient commercial plane—will determine the best choice of the parameters.

VI. NONIDEAL COMPRESSOR AND TURBINE

Ideal jet engines do not exist: Some energy is lost in all its components. More interestingly, even if these losses are neglected, irreversibilities will affect engine performance. Suppose, for example, that almost no thermal energy is supplied to the system. In this case, how can the compressor use the work generated by the turbine to compress the air in the same way as if more thermal energy were available?

We consider here a more realistic compressor and turbine. The effect of irreversibilities is introduced through the isentropic efficiencies, respectively, η_c and η_t, which compare the work consumed or produced in an adiabatic irreversible process to that involved in an ideal (reversible) process,

$$\eta_c = \frac{w_s}{w_r} = \frac{T_{2s} - T_1}{T_{2r} - T_1}, \qquad \eta_t = \frac{w_r}{w_s} = \frac{T_{4r} - T_3}{T_{4s} - T_3}. \tag{26}$$

The subscripts s and r denote the ideal (isentropic) and real transformations, and we have used energy conservation [Eq. (8)]. These efficiencies are typically of the order of 70%–90%. Irreversibilities directly affect the thermal efficiency η_{th}, which in turn affects the propulsive efficiency and the reduced thrust [see Eqs. (22) and (23)]; $\widetilde{T}_q$ remains the same.

If we introduce these new parameters and recalculate the exit temperature, we obtain the complicated expression

$$\frac{T_e}{T_i} = \frac{a[\widetilde{T}_q + (1 + \epsilon)][(1 + \epsilon)(1 + a(\eta_c - 1)) + a\widetilde{T}_q\eta_c]\eta_t}{(1 + \epsilon)[a\widetilde{T}_q\eta_c\eta_t + (1 + \epsilon)(\eta_t - 1 + a(1 + (\eta_c - 1)\eta_t))]}. \tag{27}$$

We can see why most (if not all) textbooks on thermodynamics do not discuss this result and instead discuss the nonideal jet engine as a numerical example.

To obtain the final thermal efficiency, we have to use the expression for T_e/T_i in Eqs. (9) and (5). The quantity η_{th} depends not only on the compression ratio r (or a) and the reduced kinetic energy ϵ but also on the isentropic efficiencies η_c and η_t. More interestingly and unlike in the ideal case, the thermal efficiency of the irreversible jet engine now depends on the absorbed thermal energy q (or $\widetilde{T}_q$). Nevertheless, some simple results can be derived for limiting situations. If the absorbed energy is very large ($\widetilde{T}_q \gg 1$), we recover the ideal thermal efficiency,

$$\lim_{q \to \infty} \eta_{th} = 1 - \frac{a}{1 + \epsilon}. \tag{28}$$

As the absorbed energy decreases, so does the thermal efficiency. Eventually, if the absorbed energy is sufficiently small, the work generated by the irreversible turbine is no longer able to activate the compressor, and the engine does not work (unless the air decelerates).

In Fig. 6 we show the overall efficiency η for two maximum temperatures, $T_3/T_i = 5$ and $T_3/T_i = 3$, the same compressor ratios as before, $a = 0.7$ ($r \approx 3$) and $a = 0.4$ ($r \approx 30$), and different isentropic efficiencies $\eta_c = \eta_t = 1 - z$, as a function of the dimensionless velocity M. To have a positive absorbed energy, recall that $\epsilon < aT_3/T_i - 1$ [see Eq. (25)].

As expected, the overall efficiency diminishes with increasing irreversibility z, especially for large compression ratios (light gray curves). This effect is even more pronounced for lower ($T_3/T_i = 3$) maximum temperatures.

VII. DISCUSSION

We derived a simple analytical expression for the thermal efficiency of a jet engine [Eq. (20)]. An analysis of this result shows why the invention of turbojet engines is regarded as a technological revolution in aircraft engine manufacturing. We calculated two performance indicators of jet engines, the overall efficiency and the reduced thrust, and showed that engines with better efficiencies (used mostly in commercial planes) do not usually exhibit the best thrust capacities (usually required in military aircraft). Optimization of these performances is difficult and depends, among other things, on the maximum velocity we require the aircraft to achieve.
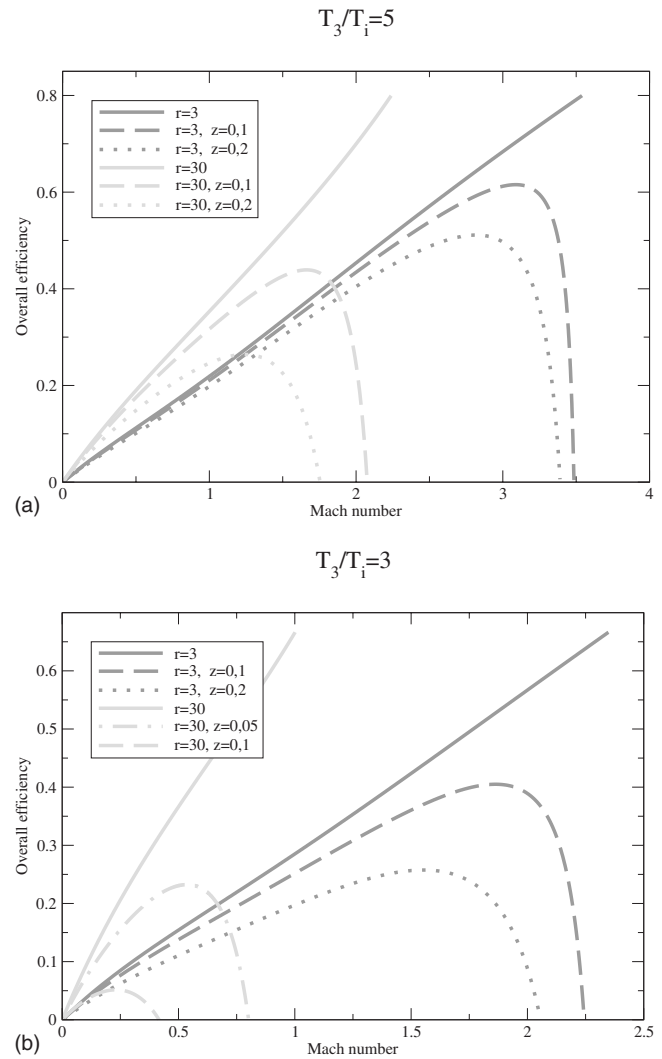

Fig. 6. Constrained efficiencies of the jet engine.

We did not discuss other major technological achievements of jet engines. One example is the afterburner, which is placed between the turbine and the nozzle and in which fuel is again injected, thereby creating a new combustion chamber and increasing aircraft thrust. Because they consume more fuel, afterburners are not efficient and for that reason are commonly used only in military aircraft. Another technological advance is turbofan engines.[6,7] Here, a large fan driven by the turbine forces a considerable amount of air through a duct surrounding the engine. The ratio of the airflow mass rate bypassing the combustion chamber, $\dot{m}_d$, to that of the air flowing through it, $\dot{m}_c$, is called the bypass ratio BPR = $\dot{m}_d/\dot{m}_c$ and is typically around 5–6. Turbofans reduce fuel consumption considerably and are responsible for the success of jumbo planes, which carry a few hundred people at speeds of almost 1000 km/h. New engines, called propjets, achieve still higher efficiencies, with bypass ratios of the order of 100.

The detailed analysis of jet engines is complex and ranges from the description of engines to the aerodynamical or structural problems associated with aircraft (see Refs. 11 and 12, for example). Our numerical analysis can be extended to other interesting problems, such as finding, for fixed M and T_3/T_i, which value of r maximizes the thrust and calculating

the effects of irreversibilities on thermal efficiency and reduced thrust. We hope that our article arouses the curiosity of readers and encourage them to undertake studies of this fascinating subject.

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[a])Electronic mail: pedro.patricio@dem.isel.pt

[1]P. M. Fishbane, S. Gasiorowicz, and S. T. Thornton, *Physics for Scientists and Engineers*, 2nd ed. (Prentice-Hall, Upper Saddle River, NJ, 1996).

[2]Y. A. Çengel and M. A. Boles, *Thermodynamics: An Engineering Approach*, 3rd ed. (McGraw-Hill, New York, 1998).

[3]M. J. Moran and H. N. Shapiro, *Fundamentals of Engineering Thermodynamics*, 5th ed. (Wiley, Hoboken, NJ, 2006).

[4]This expression is valid when the inlet air pressure is equal to its exit pressure, an assumption that we will use throughout the article.

[5]We will assume that the mass fuel rate $\dot{m}_f \ll \dot{m}$.

[6]R. D. Flack, *Fundamentals of Jet Propulsion with Applications* (Cambridge U. P., New York, 2005).

[7]W. H. Heiser and D. T. Pratt, *Hypersonic Airbreathing Propulsion* (AIAA, Reston, VA, 1994).

[8]Z. S. Spakowszky, "Thermodynamics and propulsion," ⟨web.mit.edu/16.unified/www/SPRING/propulsion/notes/notes.html⟩.

[9]In the combustion chamber, conservation of energy yields $\delta q = dh$ for any elementary transformation. The elementary variation of specific entropy may be written as $ds = dh/T - v/T dP = \delta q/T - v/T dP$ (here, v is the specific volume). If the transformation is reversible, we also have $ds = \delta q/T$, implying no pressure variation, $dP = 0$. It is possible to see from this argument that any increase of entropy due to irreversibilities will lead to a pressure drop, $ds_{\mathrm{irrev}} = -v/T dP > 0$.

[10]It is surprising that this result is rarely mentioned in textbooks. The thermal efficiency of a jet engine is sometimes given as an example of the application of the Brayton cycle, which consists of two isentropic and two isobaric transformations. This closed cycle can mimic the thermodynamics of airflow in the jet engine: An isentropic compression from the inlet to the combustor, an isobaric heating in the combustion chamber, an isentropic expansion in the turbine and in the nozzle, and an (artificial) isobaric cooling of the air back to the inlet. It is also common to express the efficiency of this cycle in terms of the ratio of the pressure at the inlet P_i to the pressure in the combustion chamber. This analysis does not split the increase in pressure into contributions from the inlet (dependent on v_i or ϵ) and from the compressor (dependent on r or a). Moreover, as in the usual analysis of closed cycles, the aim is to produce work, but we know that in jet engines, the aim is to produce thrust. Therefore, direct application of the analysis of the Brayton cycle to the jet engine yields a very limited description.

[11]J. D. Anderson, Jr., *Fundamentals of Aerodynamics*, 2nd ed. (McGraw-Hill, Singapore, 1991).

[12]T. H. G. Megson, *Aircraft Structures for Engineering Students*, 4th ed. (Elsevier, Oxford, 2008).

Capacity Bridge. This Type 216 Capacity Bridge is listed in the 1935 General Radio Company catalogue for $175. It is a specialized alternating-current bridge that was designed to find the value of fairly small capacitors. The capacitance standard, the 1000 Hz signal source and the detector (earphones with amplification) are external to the system. Note that interior of the bridge is lined with sheet copper for shielding. The instrument came to the Greenslade Collection from Wellesley College. (Photograph and Notes by Thomas B. Greenslade, Jr., Kenyon College)